Nonlinear Analysis on Flutter of Functional Graded Cylindrical Panels on Elastic Foundations Using the Ilyushin Nonlinear Supersonic Aerodynamic Theory

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Abstract: Based on classical shell theory with the geometrical nonlinearity in von Karman-Donell sense and the Ilyushin nonlinear supersonic aerodynamic theory, this paper successfully formulated the equations of motion of the functionally graded cylindrical panel on elastic foundations under impact of a moving supersonic airflow and found the critical velocity of supersonic airflow that make the panel unstable. This paper also used the Bubnov-Galerkin and Runge – Kutta methods to solve the system of nonlinear vibration differential equations and illustrated effects of initial dynamical conditions, shape and geometrical parameters, material constituents and elastic foundations on aerodynamic response and instability of FGM cylindrical panel.

Keywords: Nonlinear flutter, the Ilyushin supersonic aerodynamic theory, functional graded cylindrical panel, elastic foundations.

1. Introduction

Functionally Graded Materials (FGMs) are composite materials which have mechanical properties varying smoothly from one surface to other surface of structure. The concept of functionally graded material was proposed in 1984 [1]. Due to functionally graded materials have many advantaged properties more than common materials such as: high carrying capacity, high temperature endurance,… therefore, functionally graded materials often are used in shipbuilding industry, heat-resistance structures, aerospace and elements in nuclear reactors [2].

Moreover, today functionally graded materials are widely used in structures flying at the supersonic speed such as: wings of aircraft, spacecraft, rockets,… With the structures in such a supersonic speed, the investigation about stability of structures to guarantee and enhance safety of structures is very important.

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When the structures operated in high speed conditions, they often occur instability and self-excited vibrations tending to oscillate seriously and destroy structures, this phenomena is used to call "flutter". The issue needed for research is to find out the maximum value of velocity in which the structure still can stand in process to minimize the happening of flutter phenomenon and identify the range of velocity in which the structure is working stably, so that it can avoid problems with the structures and equipment mentioned above.

The nonlinear flutter of structures under impact of high-speed airflow have been studied by a number of researchers such as the study of Ibrahim et al. [3] about thermal buckling and nonlinear flutter behavior of FGM panels, the study of Sohn et al. [4] about using first-order shear deformation theory with the nonlinearity geometrical in von Karman and first order piston theory to investigate the nonlinear thermal flutter of functionally graded panels under a supersonic airflow and Newton-Raphson method is adopted to obtain approximate solutions of the nonlinear governing equations. Prakash et al. [5] investigated the large amplitude flexural vibration characteristics of FGM plates under aerodynamic load, the FGM plate is modeled using the first-order shear deformation theory based on exact neutral surface position and von Karman's assumption for large displacement, the third-order piston theory is employed to evaluate the aerodynamic pressure. Prakash and Ganapathi [6] used first-order shear deformation theory and first-order high Mach number including effects of temperature to investigate the supersonic flutter behavior of flat panels made of functionally graded materials under impact of supersonic airflow. Ganapathi et al. [7] studied the flutter behavior of composite panel subjected to thermal stress. By using Love's shell theory and von Karman-Donnell-type of kinematic nonlinearity coupled with linearized first-order potential Haddadpour et al. [8] studied the supersonic flutter prediction of functionally graded cylindrical shells. Based on Lagrange's equations of motion and the first-order high Mach number approximation to potential linear flow theory, Singha et al. [9] investigated the supersonic flutter behavior of laminated composite skew flat panels. Moon et al. [10] studied suppression of nonlinear composite panels flutter with active/ passive hybrid piezoelectric networks by using finite element method and the governing equations of the electromechanical coupled composite panel flutter are derived through an extended Hamilton's principle. The supersonic/ hypersonic flutter and post-flutter of geometrically imperfect circular cylindrical panels was studied by Librescu et al. [11].

However, up to date, there is no publication that carried out the nonlinear flutter of FGM panels by using Ilyushin supersonic aerodynamic theory [12]. The Ilyushin supersonic aerodynamic theory was used in the works of Stepanov [13] and Oghibalov [14] for investigating supersonic flutter behavior of isotropic plates lying in the moving supersonic airflow.

With combination of classical shell theory with nonlinearity geometrical in von Karman-Donell and supersonic aerodynamic theory of A.A.Ilyushin, in this paper, we established the governing equations to investigate nonlinear flutter behavior of FGM cylindrical panel resting on elastic foundations. The influences of nonlinear elastic foundations, initial geometrical parameters and constituent materials on critical velocities and dynamic response of the FGM panels are considered.

2. Governing equations

Consider a functionally graded cylindrical panel with radius of curvature R, axial length a, arc length b and it is defined in coordinate system (x, θ, z), where x and θ are in the axial and circumferential directions of the panel respectively and z is perpendicular to the middle surface and points inward $(-h/2 \le z \le h/2)$. In this paper, the panel is considered with large shallowness and setting $y = R\theta$ in the new coordinate (Fig. 1).
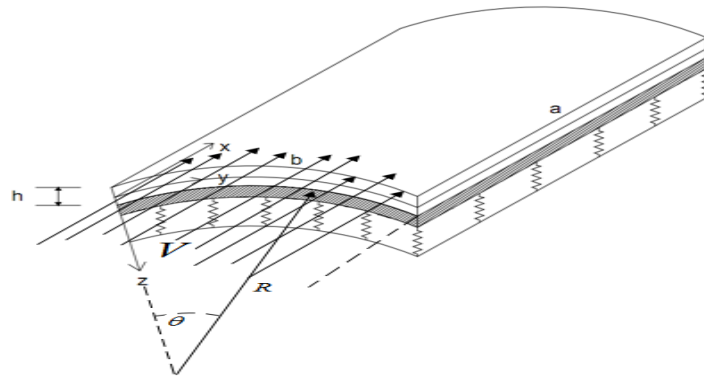

Fig. 1. The concept of FGM cylindrical panel resting on elastic foundations lying in the moving supersonic airflow.

Specific expressions of modulus of elasticity E and the mass density ρ are obtained by

$$E(z) = E_m + E_{cm}\left(\frac{2z+h}{2h}\right)^k, \ \rho(z) = \rho_m + \rho_{cm}\left(\frac{2z+h}{2h}\right)^k, \quad (1)$$

where N is volume fraction index $(0 \le N < \infty)$, m and c stand for the metal and ceramic constituents; $E_{cm} = E_c - E_m$, $\rho_{cm} = \rho_c - \rho_m$ and the Poisson's ratio v is assumed constant.

According to the classical shell theory and geometrical nonlinearity in von Karman sense, the strain across the panel thickness at the distance z from the middle surface are [15]

$$\left(\varepsilon_x \varepsilon_y, \varepsilon_z\right) = \left(\varepsilon_x^0, \varepsilon_y^0, \varepsilon_z^0\right) + z\left(\chi_x, \chi_y, 2\chi_{xy}\right). \quad (2)$$

The strains at the middle surface and curvatures of the panel as [15]

$$\varepsilon_x^0 = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2, \ \chi_x = \frac{\partial^2 w}{\partial x^2},$$

$$\varepsilon_y^0 = \frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 - \frac{w}{R}, \ \chi_y = \frac{\partial^2 w}{\partial y^2}, \quad (3)$$

$$\gamma_{xy}^0 = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y}, \ \chi_{xy} = \frac{\partial^2 w}{\partial x \partial y},$$

The force and moment resultants of the FGM panel are determined by

$$
\begin{aligned}
\left(N_x, M_x\right) &= \frac{1}{1-v^2}[\left(E_1, E_2\right)\left(\varepsilon_x^0 + v\varepsilon_y^0\right) + \left(E_2, E_3\right)\left(\chi_x + v\chi_y\right)], \\
\left(N_y, M_y\right) &= \frac{1}{1-v^2}[\left(E_1, E_2\right)\left(\varepsilon_y^0 + v\varepsilon_x^0\right) + \left(E_2, E_3\right)\left(\chi_y + v\chi_x\right)], \\
\left(N_{xy}, M_{xy}\right) &= \frac{1}{2(1+v)}[\left(E_1, E_2\right)\gamma_{xy}^0 + 2\left(E_2, E_3\right)\chi_{xy}],
\end{aligned}
\tag{4}
$$

where

$$
E_1 = \left(E_m + \frac{E_{cm}}{k+1}\right)h, \ E_2 = \frac{E_{cm}kh^2}{2(k+1)(k+2)}, \ E_3 = \left[\frac{E_m}{12} + E_{cm}\left(\frac{1}{k+3} - \frac{1}{k+2} + \frac{1}{4(k+1)}\right)\right]h^3. \tag{5}
$$

The aerodynamic pressure load q is be determined as [12]

$$
-q = B\frac{\partial w}{\partial t} - BV\frac{\partial w}{\partial x} - 2B_1V\frac{\partial w}{\partial t}\frac{\partial w}{\partial x} + B_1V^2\left(\frac{\partial w}{\partial x}\right)^2 \tag{6}
$$

and p_∞, V_∞ the pressure and the sound velocity of the quiet airflow (not excited), V is the airflow velocity on the surface structure, ζ is the Politrop index.

The nonlinear motion equation of the FGM cylindrical panels based on classical shell theory are given by Brush and Almroth [15] using Volmir's assumption [16] as

$$
\begin{aligned}
&\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0, \\
&\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = 0, \\
&\frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x\partial y} + \frac{\partial^2 M_y}{\partial y^2} + N_x\frac{\partial^2 w}{\partial x^2} + 2N_{xy}\frac{\partial^2 w}{\partial x\partial y} + N_y\frac{\partial^2 w}{\partial y^2} + \frac{1}{R}N_y \\
&+ q - K_1\mathrm{w} + K_2\Delta\mathrm{w} = \rho_1\frac{\partial^2 \mathrm{w}}{\partial t^2},
\end{aligned}
\tag{7}
$$

with $\rho_1 = \left(\rho_c + \frac{\rho_{mc}}{N+1}\right)h$ and K_1, K_2 are stiffness of Winkler and Pasternak foundation.

Putting Eq. (4) into Eq. (7) we obtain

$$
\begin{aligned}
&\rho_1\frac{\partial^2 w}{\partial t^2} + B\frac{\partial w}{\partial t} - BV\frac{\partial w}{\partial x} - 2B_1V\frac{\partial w}{\partial t}\frac{\partial w}{\partial x} + B_1V^2\left(\frac{\partial w}{\partial x}\right)^2 + D\Delta\Delta w - \frac{1}{R}\frac{\partial^2 f}{\partial x^2} + 2\frac{\partial^2 f}{\partial x\partial y}\frac{\partial^2 w}{\partial x\partial y} \\
&- \frac{\partial^2 f}{\partial y^2}\frac{\partial^2 w}{\partial x^2} - \frac{\partial^2 f}{\partial x^2}\frac{\partial^2 w}{\partial y^2} + K_1\mathrm{w} - K_2\Delta\mathrm{w} = 0,
\end{aligned}
\tag{8}
$$

where $f(x, y)$ is stress function defined by $N_x = \dfrac{\partial^2 f}{\partial y^2}$, $N_y = \dfrac{\partial^2 f}{\partial x^2}$, $N_{xy} = -\dfrac{\partial^2 f}{\partial x \partial y}$,

(9)

and $D = \dfrac{E_3}{(1 - v^2)}$

The geometrical compatibility equation for a cylindrical panel is written as

$$\frac{1}{E_1} \Delta \Delta f = \left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 - \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} - \frac{1}{R} \frac{\partial^2 w}{\partial x^2}.$$

(10)

The couple of Eqs. (8) and (10) are governing equations to investigate the nonlinear flutter of the FGM panel using the Ilyushin supersonic aerodynamic theory.

3. Solution of the problem

In the present study, the edges panels are assumed to be simply supported and freely movable. Depending on an in-plane restrain at the edges, the boundary conditions are

$$w = N_{xy} = M_x = 0, N_x = 0, N_{xy} = 0, at \ x = 0, a,$$
$$w = N_{xy} = M_y = 0, N_y = 0, N_{xy} = 0, \ at \ y = 0, b.$$

(11)

The approximate two-terms Fourier expansion solution can be written as

(12)

$$w = W_1 \sin \frac{\pi x}{a} \sin \frac{\pi y}{b} + W_2 \sin \frac{2\pi x}{a} \sin \frac{\pi y}{b},$$

where $W_1(t)$ and $W_2(t)$ are time dependent functions.

Substituting Eq. (12) into the compatibility Eq. (10), the stress function can be defined as

$$f = F_1 cos\frac{2\pi y}{b} + F_2 cos\frac{2\pi x}{a} + F_3 cos\frac{4\pi x}{a} + F_4 cos\frac{2\pi x}{a} cos\frac{\pi x}{a} +$$
$$+ F_5 cos\frac{2\pi y}{b} cos\frac{2\pi x}{a} cos\frac{\pi x}{a} + F_6 \sin\frac{2\pi x}{a} \sin\frac{\pi x}{a} + F_7 cos\frac{2\pi y}{b} \sin\frac{2\pi x}{a} \sin\frac{\pi x}{a} +$$
$$+ F_8 \sin\frac{\pi x}{a} \sin\frac{\pi y}{b} + F_9 \sin\frac{2\pi x}{a} \sin\frac{\pi y}{b} + \frac{1}{2} N_{0x} y^2 + \frac{1}{2} N_{0y} x^2,$$

(13)

in which

$$F_1 = \frac{E_1 b^2}{32a^2} \left(W_1^2 + 4W_2^2 \right); \ F_2 = \frac{E_1 a^2}{32b^2} W_1^2; \ F_3 = \frac{E_1 a^2}{128b^2} W_2^2,$$

$$F_4 = -\frac{2}{9} \frac{E_1 a^2}{b^2} W_1 W_2; \ F_5 = \frac{2E_1 a^2 b^2 \left(16a^4 + 80a^2 b^2 + 91b^4 \right)}{81b^8 + 720a^2 b^6 + 1888a^4 b^4 + 1280a^6 b^2 + 256a^8} W_1 W_2,$$

$$F_6 = -\frac{5}{18}\frac{E_1 a^2}{b^2}W_1 W_2; \quad F_7 = \frac{E_1 a^2 b^2\left(328a^2 b^2 + 365b^4 + 80a^4\right)}{2\left(81b^8 + 720a^2 b^6 + 1888a^4 b^4 + 1280a^6 b^2 + 256a^8\right)}W_1 W_2,$$

$$F_8 = \frac{E_1}{\pi^2 R}\frac{a^2 b^4}{\left(a^2 + b^2\right)^2}W_1; \quad F_9 = 4\frac{E_1}{\pi^2 R}\frac{a^2 b^4}{\left(a^2 + 4b^2\right)^2}W_2. \tag{14}$$

Assume that the panel is only subjected to the impact of airflow (not by temperature and axial load), so $N_{0x} = N_{0y} = 0$.

Substituting Eqs. (12), (13) into Eq. (8) and applying Bubnov-Galerkin method to the resulting equation yields

$$m_1 \frac{\partial^2 W_1}{\partial t^2} + m_2 \frac{\partial W_1}{\partial t} + m_3 \frac{\partial W_1}{\partial t}W_2 - m_4 \frac{\partial W_2}{\partial t}W_1 +$$

$$+ m_5 W_2 + m_6 W_2^2 + m_7 W_1 + m_8 W_1^2 + m_9 W_1 W_2^2 + m_{10} W_1^3 = 0, \tag{15}$$

$$l_1 \frac{\partial^2 W_2}{\partial t^2} + l_2 \frac{\partial W_2}{\partial t} - l_3 \frac{\partial W_1}{\partial t}W_1 - l_4 W_1 + l_5 W_1 W_2 + l_6 W_2 + l_7 W_1^2 W_2 + l_8 W_2^3 = 0, \tag{16}$$

where

$$m_1 = \rho_1 \frac{ab}{4}, \quad m_2 = B\frac{ab}{4}, \quad m_3 = \frac{4}{3}B_1 V b, \quad m_4 = \frac{2}{3}bB_1 V,$$

$$m_5 = \frac{2}{3}bBV, \quad m_6 = \frac{224}{45}\frac{b}{a}B_1 V^2 - \frac{1}{30}\frac{aE_1}{bR} - \frac{128}{5}\frac{ab^3 E_1}{R(4b^2 + a^2)^2},$$

$$m_7 = \frac{D\pi^4}{4}\left(\frac{b}{a^3} + \frac{2}{ab} + \frac{a}{b^3}\right) + \frac{ab}{4}\left(K_1 + K_2\left(\frac{\pi^2}{a^2} + \frac{\pi^2}{b^2}\right)\right) + \frac{E_1}{4}\frac{ab^5}{R^2(a^2 + b^2)^2},$$

$$m_8 = \frac{8}{9}\frac{b}{a}B_1 V^2 - \frac{E_1}{6}\frac{a}{bR} - \frac{8}{3}\frac{E_1 ab^3}{R(a^2 + b^2)^2},$$

$$m_9 = \frac{1}{2}\frac{E_1 ab\pi^4\left(16a^4 + 80a^2 b^2 + 91b^4\right)}{\left(81b^8 + 720a^2 b^6 + 1888a^4 b^4 + 1280a^6 b^2 + 256a^8\right)}$$

$$+ \frac{5E_1 ab\pi^4\left(328a^2 b^2 + 365b^4 + 80a^4\right)}{32\left(81b^8 + 720a^2 b^6 + 1888a^4 b^4 + 1280a^6 b^2 + 256a^8\right)} + \frac{\pi^4 E_1}{16}\left(\frac{a}{b^3} + \frac{b}{a^3}\right), \tag{17}$$

$$m_{10} = \frac{\pi^4 E_1}{64}\left(\frac{b}{a^3} + \frac{a}{b^3}\right).$$

$$l_1 = \frac{\rho_1 ab}{4}, \quad l_2 = \frac{Bab}{4}, \quad l_3 = \frac{2}{3}bB_1 V, \quad l_4 = \frac{2}{3}bBV,$$

$$l_5 = \frac{64b}{45a}B_1 V^2 - \frac{16}{15}\frac{aE_1}{bR} - \frac{32}{5}\frac{E_1 ab^3}{R(a^2 + b^2)^2} - \frac{128}{5}\frac{E_1 ab^3}{R(a^2 + 4b^2)^2}$$

$$+\frac{176}{45}\frac{E_1 ab^3\left(16a^4+80a^2b^2+91b^4\right)}{R\left(81b^8+720a^2b^6+1888a^4b^4+1280a^6b^2+256a^8\right)}$$

$$-\frac{128E_1 ab^3\left(328a^2b^2+365b^4+80a^4\right)}{90R\left(81b^8+720a^2b^6+1888a^4b^4+1280a^6b^2+256a^8\right)},$$

$$l_6=\frac{D\pi^4}{4}(16\frac{b}{a^3}+\frac{8}{ab}+\frac{a}{b^3})+\frac{ab}{4}K_1+(\frac{\pi^2 b}{a}+\frac{\pi^2 a}{4b})K_2+\frac{4ab^5 E_1}{R^2(a^2+4b^2)^2},$$

$$l_7=\frac{1}{2}\frac{E_1 ab\pi^4\left(16a^4+80a^2b^2+91b^4\right)}{\left(81b^8+720a^2b^6+1888a^4b^4+1280a^6b^2+256a^8\right)}$$

$$+\frac{5E_1 ab\pi^4\left(328a^2b^2+365b^4+80a^4\right)}{32\left(81b^8+720a^2b^6+1888a^4b^4+1280a^6b^2+256a^8\right)}+\frac{\pi^4 E_1}{16}(\frac{a}{b^3}+\frac{b}{a^3}),$$

$$l_8=\frac{\pi^4 E_1}{64}(\frac{a}{b^3}+16\frac{b}{a^3}).$$

Setting $\tau=\dfrac{V_\infty t}{a}$; $\phi_1=\dfrac{W_1}{h}$; $\phi_2=\dfrac{W_2}{h}$ to Eqs. (15) and (16), after some rearrangements, obtained equations may be written in the following form

$$\frac{\partial^2\phi_1}{\partial\tau^2}+M_2\frac{\partial\phi_1}{\partial\tau}+M_3\frac{\partial\phi_1}{\partial\tau}\phi_2-M_4\frac{\partial\phi_2}{\partial\tau}\phi_1+M_5\phi_2+M_6\phi_2^2+M_7\phi_1$$

$$+M_8\phi_1^2+M_9\phi_1\phi_2^2+M_{10}\phi_1^3=0,$$
(18)

$$\frac{\partial^2\phi_2}{\partial^2\tau}+L_2\frac{\partial\phi_2}{\partial\tau}-L_3\frac{\partial\phi_1}{\partial\tau}\phi_1-L_4\phi_1+L_5\phi_1\phi_2+L_6\phi_2+L_7\phi_1^2\phi_2+L_8\phi_2^3=0$$
(19)

where denote

$$M_2=\frac{a}{V_\infty}\frac{m_2}{m_1},\ M_3=\frac{ah}{V_\infty}\frac{m_3}{m_1},\ M_4=\frac{ah}{V_\infty}\frac{m_4}{m_1},\ M_5=\left(\frac{a}{V_\infty}\right)^2\frac{m_5}{m_1},$$

$$M_6=\frac{h}{m_1}\left(\frac{a}{V_\infty}\right)^2 m_6,\ M_7=\frac{\left(\frac{a}{V_\infty}\right)^2 m_7}{m_1},\ M_8=\frac{h\left(\frac{a}{V_\infty}\right)^2 m_8}{m_1},\ M_9=\frac{\left(\frac{ah}{V_\infty}\right)^2 m_9}{m_1},\ M_{10}=\frac{\left(\frac{ah}{V_\infty}\right)^2 m_{10}}{m_1},$$
(20)

$$L_2=\frac{a}{V_\infty}\frac{l_2}{l_1},\ L_3=\frac{ah}{V_\infty}\frac{l_3}{l_1},\ L_4=\left(\frac{a}{V_\infty}\right)^2\frac{l_4}{l_1},\ L_5=\frac{h\left(\frac{a}{V_\infty}\right)^2 l_5}{l_1},$$

$$L_6=\frac{\left(\frac{a}{V_\infty}\right)^2 l_6}{l_1},\ L_7=\frac{\left(\frac{ah}{V_\infty}\right)^2 l_7}{l_1},\ L_8=\frac{\left(\frac{ah}{V_\infty}\right)^2 l_8}{l_1}.$$

System of second-order differential equations with non-dimensional coefficients (18), (19) can be used to investigate the nonlinear flutter of FGM cylindrical panel. It's very difficult to find out the exact solutions of these equations, therefore, we will solve this system of differential equations by using the fourth-order Runge-Kutta procedure with some cases of initial conditions.

4. Numerical results and discussion

4.1. Validation

This section compares obtained result with other result using A.A.Ilyushin's aerodynamic theory in order to illustrate the similarity between two investigations and to increase the reliability of this approach. The material's parameters of the isotropic plate are chosen as [13,14]

$$\eta = 1.4, \ E = 2\times10^6(\frac{kg}{cm^2}), \ \rho_0 = 7.8\times10^{-3}(\frac{kg}{cm^3}), \ p_\infty = 1.014(\frac{kg}{cm^2}), \ V_0 = V_\infty = 3.4\times10^4(\frac{cm}{s}).$$

From Fig. 2 up to Fig.5 show the similarity in the obtained results of this study with Oghibalov's results for the isotropic plate [14] (cylindrical panel becomes plate with $R \rightarrow \infty$) in cases of instability (Fig. 2 and Fig. 3) and stability (Fig. 4 and Fig. 5).
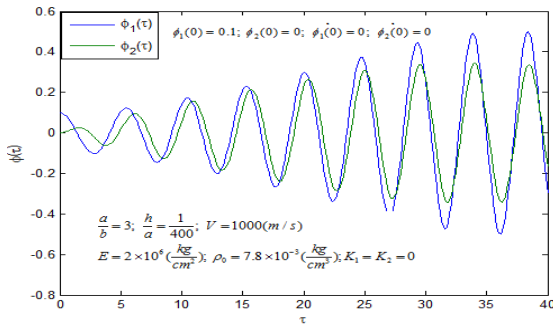
Fig. 2. The present result in case of instability of the plate.
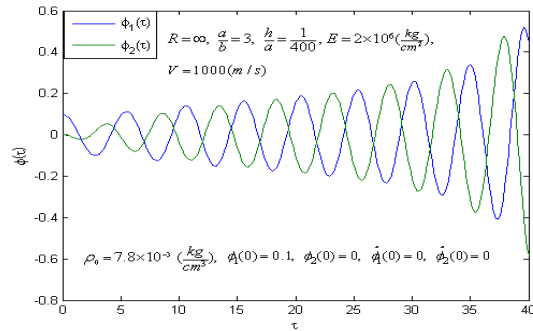
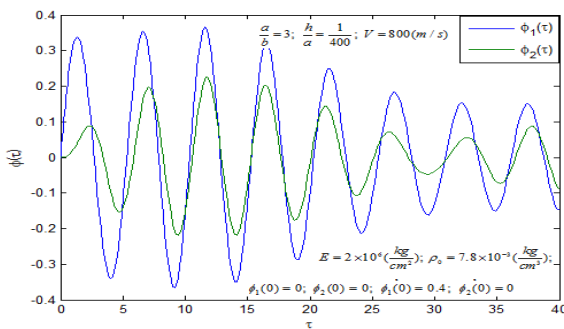Fig. 3. The Oghibalov's result [14] in case of instability of the plate.

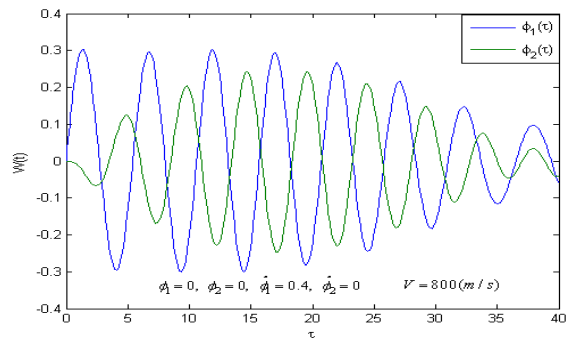Fig. 4. The present result in the case of stability of the plate.

Fig. 5. The Oghibalov's result [14] in case of stability of the plate.

4.2. Nonlinear flutter of FGM cylindrical panels on elastic foundations

In this section, we will investigate the nonlinear flutter of the FGM cylindrical panel with different initial conditions by considering response of the panel in each specific case and from which finding out the features of instability of the panel. The data of materials, geometrical parameters and aerodynamic conditions are as following

$\eta = 1.4, \ V_0 = V_\infty = 340(m / s), \ p_\infty = 1.014 \times 10^5 (Pa)$

$E_c = 380 \ (GPa), E_m = 70 \ (GPa), \rho_c = 3800 \ (kg / m^3),$

$\rho_c = 2702 \ (kg / m^3), \ v = 0.3$

Case 1: $\begin{cases} (1a) \ \phi_1(0) = 0.1, \ \phi_2(0) = 0, \ \dot{\phi}_1(0) = 0, \ \dot{\phi}_2(0) = 0 \\ (1b) \ \phi_1(0) = 0.1, \ \phi_2(0) = -0.1, \ \dot{\phi}_1(0) = 0, \ \dot{\phi}_2(0) = 0 \end{cases}$

Case 2: $\begin{cases} (2a) \ \phi_1(0) = 0, \ \phi_2(0) = 0, \ \dot{\phi}_1(0) = 0.1, \ \dot{\phi}_2(0) = 0 \\ (2b) \ \phi_1(0) = 0, \ \phi_2(0) = 0, \ \dot{\phi}_1(0) = 0.4, \ \dot{\phi}_2(0) = 0 \end{cases}$

Case 3: $\begin{cases} \phi_1(0) = 0.1, \ \phi_2(0) = 0, \ \dot{\phi}_1(0) = 0, \ \dot{\phi}_2(0) = 0, \\ \dfrac{h}{a} = \dfrac{1}{360}; \ \dfrac{1}{400}; \ \dfrac{1}{440}. \end{cases}$

Case 4: $\begin{cases} \phi_1(0) = 0.1, \ \phi_2(0) = 0, \ \dot{\phi}_1(0) = 0, \ \dot{\phi}_2(0) = 0, \\ k = 0; \ 1; \ 2. \end{cases}$

Case 5: $\begin{cases} \phi_1(0) = 0.1, \ \phi_2(0) = 0, \ \dot{\phi}_1(0) = 0, \ \dot{\phi}_2(0) = 0, \\ K_1 = \ 0; \ 10^3; \ 10^4 (Pa / m), \ K_2 = \ 0; \ 10^3; \ 10^4 (Pa.m). \end{cases}$

From Fig. 6 to Fig. 9, we can investigate the behavior of panel in case $1a$ - (Fig. 6 and Fig. 8) and $1b$ (Fig.7 and Fig. 9). Observing Fig. 6 to Fig 7, when the panel is still stable at the velocity of $V = 800(m / s)$, we can see that the amplitude of the panel in case $1b$ is larger than one in the case $1a$. Increasing the velocity up to $V = 980(m / s)$, the oscillation of the panel (in the case $1a$) starts becoming harmonic (happens in pre-instability period). The velocity at $V = 980(m / s)$ can be seen as the critical velocity of the panel in this case. Meanwhile, in Fig.9 (in case $1b$), the panel still oscillate stably.

Similarly, Fig. 10 up to Fig. 13 illustrate the phenomenon of flutter in case 2, when the initial velocity of $\dot{\phi}_1(\tau)$ is different from zero. Comparing between 2 cases $\dot{\phi}_1(0) = 0.1$ (case $2a$) and $\dot{\phi}_1(0) = 0.4$ (case $2b$), obviously we can see that in Fig. 10 and Fig.11 the panel is stable and the oscillation amplitude in case $2b$ is much larger than the one of case $2a$. Considering the occurrence

of instability of the panel in different initial velocities (Fig.12 and Fig.13), we can see that the critical velocity of airflow in both cases $2a$ and $2b$ is at $V = 1000(m/s)$. It is recognized by the phenomenon of continuously increasing of oscillation amplitudes by time. However, the instability in case $2b$ (Fig. 13) happens stronger than the one in case $2a$ (Fig. 12) due to the fact that the initial velocity of case $2b$ is higher than the one in case $2a$.
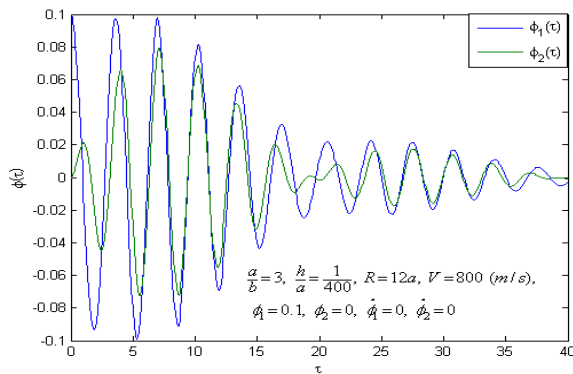

Fig. 6. Nonlinear flutter response of the FGM cylindrical panel at $V = 800(m/s)$ with initial conditions in the case $1(a)$.
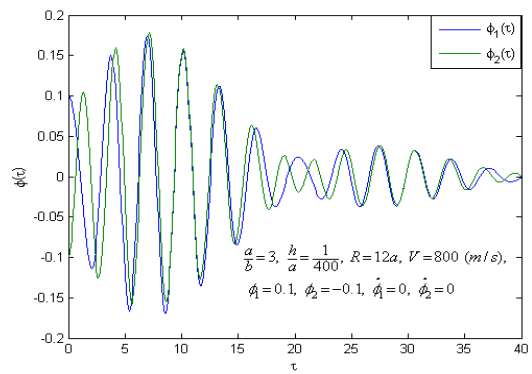
Fig. 7. Nonlinear flutter response of the FGM panel at $V = 800(m/s)$ with initial conditions in the case $1(b)$.
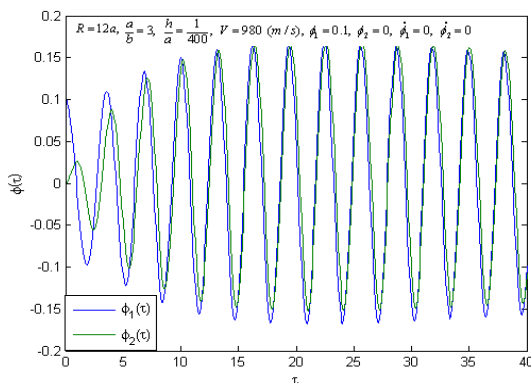
Fig. 8. Nonlinear flutter response of the FGM panel at $V = 980(m/s)$ with initial conditions in the case $1(a)$
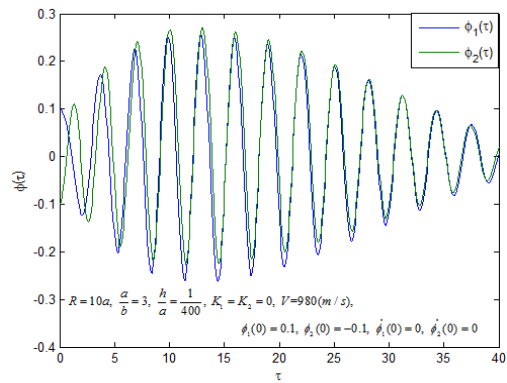
Fig. 9. Nonlinear flutter response of the FGM panel at $V = 980(m/s)$ with initial conditions in the case $1(b)$

By considering the flutter behavior of the panel from Fig. 6 up to Fig. 13, it is showing that the initial conditions affect strongly on the flutter behavior of the panel, especially the initial velocities. Therefore, we can actively control the behavior of cylindrical panel for different purposes.

Effects of geometrical dimensions on nonlinear flutter of the FGM panel are shown in Fig. 14 and Fig. 15 with initial conditions as case 3. The results from Fig. 14 show that with given airflow velocity $V = 800(m/s)$ the panel is still in the stability, although the ratio of h/a increases, the oscillation

amplitude of the panel decreases. Even the ratio of h/a does not increase much, but the oscillation amplitudes of the panel decrease much. In Fig. 15, we examine the flutter phenomenon of panels in 3 cases: $h/a = 1/360$, $h/a = 1/400$, $h/a = 1/440$ corresponding to the 3 obtained different critical velocities of airflow (the panels are instable): $V = 1385(m/s)$; $V = 1065(m/s)$; $V = 850(m/s)$. It is showing that the ratio h/a has great influence on the critical velocity: increasing the ratio h/a will increase significantly the critical velocity and decrease the oscillation amplitudes of the panel. When the h/a is getting smaller, the panel is getting thinner so the panel will be weakened due to the excitation of the airflow. Obviously, decreasing the ratio h/a will reduce the value of the flutter critical velocity, it makes the panel more easily destroyed.
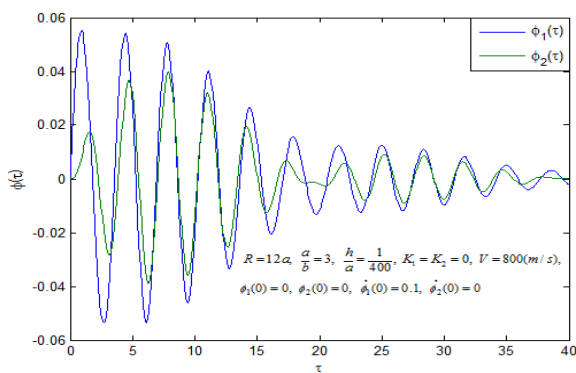
Fig. 10. Effect of initial conditions on the nonlinear flutter response of the FGM cylindrical panel in the case $2(a)$.
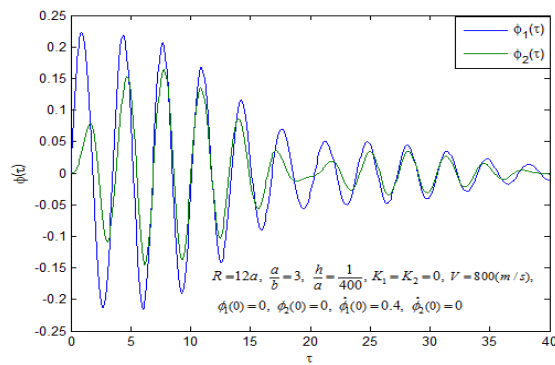

Fig. 11. Effect of initial conditions on the nonlinear flutter response of the FGM cylindrical panel in the case $2(b)$.
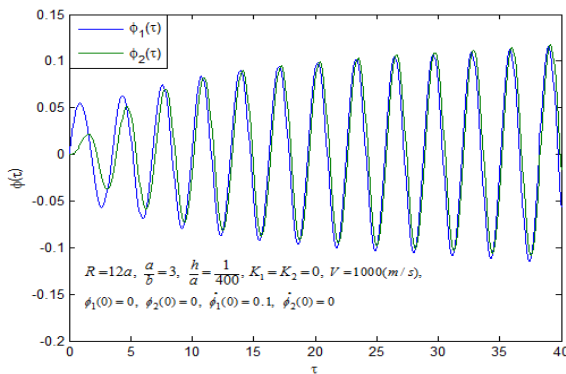

Fig. 12. Effect of initial conditions on the nonlinear flutter response of the FGM cylindrical panel at $V = 1000(m/s)$ in the case $2(a)$.
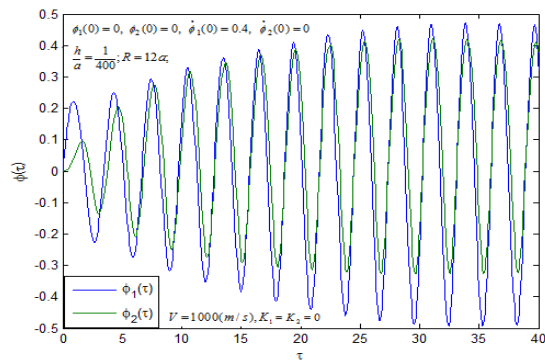

Fig. 13. Effect of initial conditions on the nonlinear flutter response of the FGM cylindrical panel at $V = 1000(m/s)$ in the case $2(b)$.
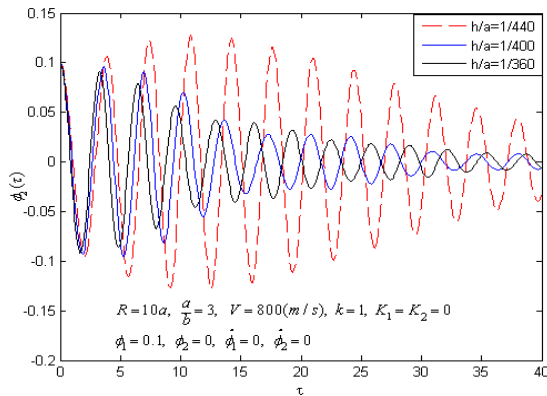
Fig. 14. Influence of the h/a ratio on the nonlinear flutter response of the FGM cylindrical panel (the panel in the stability).
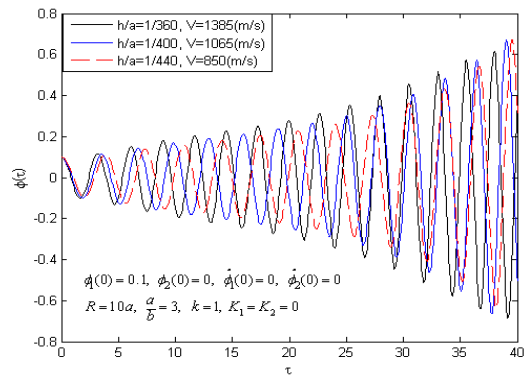

Fig. 15. Influence of the h/a ratio on the critical velocity of the FGM cylindrical panel (the panel in the instability).

The volume ratio between metal and ceramic also has a great influence on the behavior of nonlinear flutter of the panel. Specifically, the calculation is studied with the initial condition in case 4 and the results are presented in Fig. 16, when $k = 0$ (panel is made entirely from ceramic) the oscillations amplitude is quite small comparing to $k = 1$ and $k = 2$ (k increasing, the rate ceramic decreasing). Furthermore, when appears the phenomenon such as the vibration amplitude is found to increase continuously by time, the instability of FGM panel happens, and this value of velocity is called a critical flutter velocity. Fig.17 shows significant effect of volume ratio k on the critical velocities of the airflow corresponding with the large difference in three values of those coefficients: $k = 0,\ V = 1972\, m/s$ is compared with $k = 1,\ V = 1045\, m/s$ and $k = 2,\ V = 830\, m/s$. It is suitable due to the elastic module of the ceramic $\left(E_c = 380\, GPa \right)$ much larger than the metal $\left(E_m = 70\, GPa \right)$. The rich ceramic FGM panel stands much better, but this also reduces the flexibility of the panel because the ceramic is very hard but less elastic than metal.
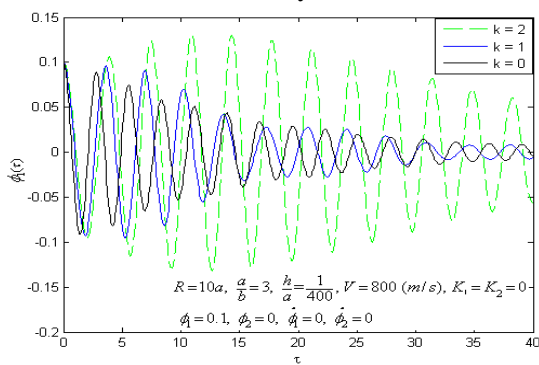

Fig. 16. Effect of volume ratio k on the nonlinear flutter response of the FGM panel.
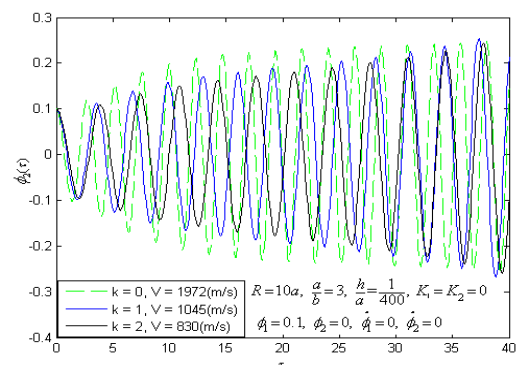

Fig. 17. Effect of volume ratio k on the critical flutter velocities of the FGM panel.

Finally, the influence of Winkler and Pasternak elastic foundations on the nonlinear flutter of FGM cylindrical panel with the initial conditions of case 5 is examined in Fig. 18. The results from Fig. 18 shows that when choosing the appropriate elastic foundations, the panel can switch from instability to stability. Obviously, when increasing the stiffness of the elastic foundations, the oscillation amplitudes of the panel will also be getting smaller correspondingly. These results show clearly the great positive influence of elastic foundations on the flutter of the panel.
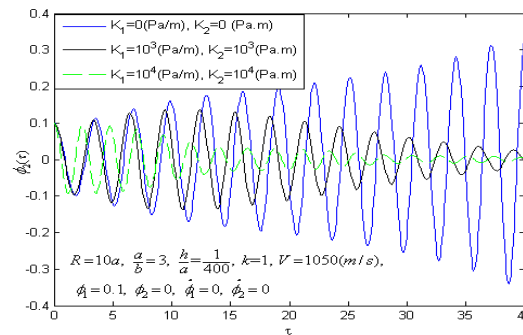

Fig. 18. Influence of Winkler and Pasternak foundations on the nonlinear flutter response of the FGM panel.

5. Conclusions

This paper established governing equations to investigate the nonlinear flutter of FGM cylindrical panels on elastic foundations under impacts of moving supersonic airflow by using the classical shell theory. We successfully formulated the equations of motion of the functionally graded cylindrical panel by the Ilyushin nonlinear supersonic aerodynamic theory and found the critical velocity of supersonic airflow that make the panel unstable.

Using Bubnov-Galerkin and Runge-Kutta methods, the paper illustrated effects of initial dynamical conditions, shape and geometrical parameters, material constituents and reinforced elastic foundations on nonlinear flutter and critical velocity. Therefore, when designing appropriately those parameters, we may actively control the flutter of FGM panels.

Acknowledgements

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