VNU Journal of Science: Mathematics – Physics, Vol. 31, No. 3 (2015) 39-48

Asymptotic Behavior of Solutions for Linear Implicit Difference Equations with Index 1

Ngo Thi Thanh Nga*

Faculty of Mathematics and Informatics, Thang Long University, Hanoi, Vietnam

Received 16 June 2015
Revised 24 July 2015; Accepted 17 August 2015

Abstract: In this paper, we deal with the asymptotic behavior of solutions of constant coefficient linear implicit difference equations with index 1. Supposing that all solutions of the original implicit equation $Ex(n+1) = Ax(n)$ are bounded (resp. tend to zero as k tends to infinity), we provide sufficient conditions imposed on the perturbations so that all solutions of the perturbed equations $(E+F(n))x(n+1) = (A+B(n))x(n)$ remain bounded (resp. tend to zero as k tends to infinity).

Key words: Implicit Difference Equations, IDEs, SDEs, Matrix Pencil, Kronecker Index

1. Introduction

In recent years, there have been many researchers interested in implicit difference equations (IDEs) (also referred to as singular difference equations, discrete-time descriptor systems) because of their appearance in many practical areas, such as the Leontiev dynamic model of multi-sector economy, the Leslie population growth model, singular discrete optimal control problems and so forth (see [1-7]). IDEs also occur naturally when we use discretization techniques for solving differential-algebraic equations (DAEs) and partial differential-algebraic equations (cf. [5, 6, 8-10]).

For the stability theory of IDEs, in [11], authors consider the stability radii for IDEs. The robust stability of implicit linear systems containing a small parameter in the leading term has been studied in [12]. However, as far as we know, there is no result considering the case where the disturbance is time-varying and arises in the leading term, too. Therefore, in this paper we deals with the preservation of asymptotic behavior of the solutions of IDEs when the perturbation is varying in time and affects both the coefficients.

The paper is organized as follows. In the next section, we summarize some basic properties of linear algebra and some results about the asymptotic behavior of solutions of linear ordinary

* Tel.: 84-1677508968
Email: nga.ngo@gmail.com

difference equations. In Section 3, we present the main result on the asymptotic behavior of the solutions of constant coefficient linear implicit difference equations with index 1. Supposing that all solutions of the original implicit equation $Ex(n+1) = Ax(n)$ are bounded (resp. tend to zero as k tends to infinity), we provide sufficient conditions imposed on the perturbations so that all solutions of the perturbed equations $\left(E+F(n)\right)x(n+1)=\left(A+B(n)\right)x(n)$ remain bounded (resp. tend to zero as k tends to infinity). Finally, we give some examples for illustration.

2. Preliminaries

In this section, we survey some basic properties of linear algebra. Let A be a $d \times d-$ matrix. The Kronecker index of the matrix A, denoted by ind A, is the smallest non-negative integer k such that $\operatorname{im} A^k = \operatorname{im} A^{k+1}$. Let $\{E, A\}$ is a regular matrix pencil, i.e., the polynomial $p(\lambda) = \det(\lambda E + A) \neq 0$. Then, the Kronecker index of the matrix pencil $\{E, A\}$, denoted by ind $\{E, A\}$, is defined as the Kronecker index of the matrix $(\lambda E + A)^{-1}E$ for λ such that $p(\lambda) \neq 0$.

Lemma 2.1 (see [12]). Let E, A be two matrices in $\mathbb{R}^{d\times d}$, with $\operatorname{rank}(E) = r$.

Suppose that the matrix pencil $\{E, A\}$ is regular. Then, there exist two invertible matrices U, V in $\mathbb{R}^{d\times d}$ such that:

$$UEV = \begin{pmatrix} E_{11} & 0 \\ 0 & 0 \end{pmatrix}, \quad UAV = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix},$$

where E_{11} is a nonsingular $r \times r$ matrix. Moreover, ind$\{E, A\} = 1$ if and only if the matrix A_{22} is nonsingular.

The matrices U and V can be constructed by the following way: let the matrices $U_1 \in \mathbb{R}^{d\times(d-r)}$ and $V_1 \in \mathbb{R}^{d\times(d-r)}$ be chosen such that their columns form (minimal) bases for the left and right null-spaces of E, respectively, i.e. $U_1^T E = 0$, $EV_1 = 0$; then we define the matrices

$$U = [U_1^{\perp} \;\; U_1]^T, \quad V = [V_1^{\perp} \;\; V_1],$$

where $U_1^{\perp}$ and $V_1^{\perp}$ are the bases of the orthogonal subspaces associated with U_1 and V_1 (see [13] for the details).

We now consider the ordinary difference equation with constant coefficient

$$x(n+1) = Ax(n), \quad n \in \mathbb{N}(n_0), \qquad (2.1)$$

where $\mathbb{N}(n_0)$ is the set of natural numbers that are greater than or equal to n_0, $x(n) \in \mathbb{R}^d$, $\forall n \in \mathbb{N}(n_0)$ and $A \in \mathbb{R}^{d\times d}$.

The perturbed equation of (2.1):

$$u(n+1) = (A + B(n))u(n), \quad n \in \mathbb{N}(n_0), \tag{2.2}$$

where $B(n) \in \mathbb{R}^{d\times d}$, $\forall n \in \mathbb{N}(n_0)$.

Denote by $x(n, n_0, x_0)$ the solution of (2.1) with the initial condition $x(n_0, n_0, x_0) = x_0$. It is easy to see that $x(n, n_0, 0) = 0$ for all $n \in \mathbb{N}(n_0)$.

Deffinition 2.2. The trivial solution $x \equiv 0$ of the difference equation (2.1) is said to be stable (for short: the system (2.1) is stable) if for any $\varepsilon > 0$, there is $\delta > 0$ such that $\|x(n, n_0, x_0)\| < \varepsilon$, for all $n \in \mathbb{N}(n_0)$ if $\|x_0\| < \delta$.

As we known, there are some important properties of the ordinary linear difference equation (2.1) (See [1]) :

- the system (2.1) is stable iff all solutions of the difference equation (2.1) are bounded on $\mathbb{N}(n_0)$. Moreover, this is equivalent to the fact that all the eigenvalues of A have modulus less than or equal to one, and those of modulus one are semisimple.
- all solutions $x(k)$ of the difference equation (2.1) tend to zero as $k \to \infty$ if and only if all the eigenvalues of the matrix A are inside the unit disc.

Theorem 2.3 (see [14]). Let all solutions of the difference equation (2.1) be bounded on $\mathbb{N}(\mathrm{n}_0)$. Then, all solutions of (2.2) are bounded on $\mathbb{N}(\mathrm{n}_0)$, provided that

$$\sum_{l=n_0}^{\infty} \|B(l)\| < \infty .$$

Theorem 2.4 (See [14]). Let all solutions of the difference equation (2.1) tend to zero as $k \to \infty$. Then, all solutions of (2.2) tend to zero as $k \to \infty$ provided $\|B(k)\| \to 0$ as $k \to \infty$.

In the paper, we will generalize two above results for the constant coefficient implicit difference equations with perturbations in both sides. Consider the linear implicit difference equation with constant coefficient

$$Ex(n+1) = Ax(n), \quad n \in \mathbb{N}(n_0), \tag{2.3}$$

where $E, A \in \mathbb{R}^{d\times d}$, rank($E$) $= r$, $x(n) \in \mathbb{R}^d$, $n \in \mathbb{N}(n_0)$.

The equation (2.3) is said to be index 1 if $\mathrm{ind}\{E, A\} = 1$. According to Lemma 2.1, there exist two invertible matrices U, V in $\mathbb{R}^{d\times d}$ such that

$$UEV = \begin{pmatrix} E_{11} & 0 \\ 0 & 0 \end{pmatrix}, \quad UAV = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix},$$

where E_{11} is a nonsingular $r \times r$ matrix and the matrix A_{22} is nonsingular, too.

Putting $x(n) = V\, y(n) = V\begin{pmatrix} y_1(n) \\ y_2(n) \end{pmatrix}$ and multiplying both sides of (2.3) by U, we obtain

$$\begin{pmatrix} E_{11} & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} y_1(n+1) \\ y_2(n+1) \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}\begin{pmatrix} y_1(n) \\ y_2(n) \end{pmatrix},$$

or

$$\begin{cases} E_{11}y_1(n+1) = A_{11}y_1(n) + A_{12}y_2(n) \\ 0 \qquad\qquad\quad = A_{21}y_1(n) + A_{22}y_2(n) \end{cases} \tag{2.4}$$

Since matrices E_{11} and A_{22} are invertible, the equation (2.4) is equivalent to the following system:

$$\begin{cases} y_1(n+1) = E_{11}^{-1}(A_{11} - A_{12}A_{22}^{-1}A_{21})y_1(n) \\ y_2(n) \quad = A_{22}^{-1}A_{21}y_1(n) \end{cases}$$

Similarly to the ordinary difference equations, we can generalize the above results to the equation (2.3). It is easy to see that all solutions of the implicit difference equation (2.3) are bounded on $\mathbb{N}(n_0)$ if and only if all the finite eigenvalues of pencil $\{E, A\}$ have modulus less than or equal to one, and those of modulus one are semi-simple. Moreover, all solutions $x(k)$ of the difference equation (2.3) tend to zero as $k \to \infty$ if and only if all the finite eigenvalues of the matrix pencil {E, A} are inside the unit disc.

3. Asymptotic behavior of solutions of linear implicit difference equations

In this section, we consider the perturbed implicit difference equation of the form

$$\left(E + F(n)\right)u(n+1) = \left(A + B(n)\right)u(n),\ n \in \mathbb{N}(n_0), \tag{3.1}$$

where $F(n), B(n) \in \mathbb{R}^{d\times d}$ are perturbations, with F is an admissible perturbations, i.e. ker E $\subset$ ker $F(n)$ or ker E $\subset$ ker $\left(E + F(n)\right)$ for all $n \in \mathbb{N}(n_0)$ (See [15]).

The following example shows that if $\ker E \not\subset \ker F(n)$ then the asymptotic behavior of solutions of the perturbed SDEs (3.1) and the asymptotic behavior of solutions of the unperturbed one may be quite different, even if the pertubation F is small, e.g., it is convergent to 0 as $l \to \infty$ and $\sum_{l=n_0}^{\infty} \|F(l)\| < \infty$.

Example 3.1. Consider the index-1 SDE

$$\begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x_1(n+1) \\ x_2(n+1) \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x_1(n) \\ x_2(n) \end{pmatrix}, \quad n \in \mathbb{N}.$$

It is easy to obtain the solution $x_1(n)=\dfrac{1}{2^n}x_1(0)$ and $x_2(n)=0$, for all $n\in\mathbb{N}$. After that, we consider the following perturbed SDE

$$\begin{pmatrix}2 & 0\\ 0 & \dfrac{1}{(n+1)^2}\end{pmatrix}\begin{pmatrix}u_1(n+1)\\ u_2(n+1)\end{pmatrix}=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}u_1(n)\\ u_2(n)\end{pmatrix}, \quad n\in\mathbb{N}. \tag{3.2}$$

From the first equation of (3.2), it follows that $u_1(n)=x_1(\mathrm{n})=\dfrac{1}{2^n}x_1(0).$

However, the second component $u_2(n)=(\mathrm{n}!)^2u_2(0),$ which tends to ∞ as $n\to\infty$. That is, a small perturbation in the leading coefficient can completely change the behavior of the solutions.

In the remainder part of this section, we assume that F is admissible. Let us apply to (3.1) the transformation with the same U and V as in Section 2 and note that in this case

$$UF(n)V=\begin{pmatrix}F_{11}(n) & 0\\ F_{21}(n) & 0\end{pmatrix},\quad UB(n)V=\begin{pmatrix}B_{11}(n) & B_{12}(n)\\ B_{21}(n) & B_{22}(n)\end{pmatrix}.$$

Putting $u(n)=V\,z(n)=V\begin{pmatrix}z_1(n)\\ z_2(n)\end{pmatrix}$ and multiplying both sides of (2.3) by U, we obtain

$$\begin{cases}\left(E_{11}+F_{11}(n)\right)z_1(n+1)=\left(A_{11}+B_{11}(n)\right)z_1(n)+\left(A_{12}+B_{12}(\mathrm{n})\right)z_2(n)\\ F_{21}(n)z_1(n+1) \qquad =\left(A_{21}+B_{21}(n)\right)z_1(n)+\left(A_{22}+B_{22}(n)\right)z_2(n)\end{cases} \tag{3.3}$$

From now on, we make the assumption

Assumption 1. Suppose that $E_{11}+F_{11}(n)$ is invertible for all $n\in\mathbb{N}(n_0)$.

If the Assumption 1 holds then

$$\left(E_{11}+F_{11}(n)\right)^{-1}=E_{11}^{-1}-E_{11}^{-1}F_{11}(n)\left(E_{11}+F_{11}(n)\right)^{-1}.$$

Multiplying the first equation of the system (3.3) by $E_{11}\left(E_{11}+F_{11}(n)\right)^{-1}$, we obtain

$$E_{11}z_1(n+1)=\left(A_{11}+\overline{B}_{11}(n)\right)z_1(n)+\left(A_{12}+\overline{B}_{12}(n)\right)z_2(n),$$

where

$$\overline{B}_{11}(n)=B_{11}(n)-F_{11}(n)\left(E_{11}+F_{11}(n)\right)^{-1}\left(A_{11}+B_{11}(n)\right)$$

$$\overline{B}_{12}(n)=B_{12}(n)-F_{11}(n)\left(E_{11}+F_{11}(n)\right)^{-1}\left(A_{12}+B_{12}(n)\right)$$

In order to bring (3.3) into the simpler form, we first multiply the first equation of (3.3) by $-F_{21}(n)\left(E_{11}+F_{11}(n)\right)^{-1}$, add the obtained result to the second equation of (3.3), we get

$$0 = \left(A_{21} + \overline{B}_{21}(n)\right) z_1(n) + \left(A_{22} + \overline{B}_{22}(n)\right) z_2(n),$$

where

$$\overline{B}_{21}(n) = B_{21}(n) - F_{21}(n)\left(E_{11} + F_{11}(n)\right)^{-1}\left(A_{11} + B_{11}(n)\right),$$

$$\overline{B}_{22}(n) = B_{22}(n) - F_{21}(n)\left(E_{11} + F_{11}(n)\right)^{-1}\left(A_{12} + B_{12}(n)\right).$$

Then, the system (3.3) is equivalent to the system

$$\begin{cases} E_{11} z_1(n+1) = \left(A_{11} + \overline{B}_{11}(n)\right) z_1(n) + \left(A_{12} + \overline{B}_{12}(n)\right) z_2(n) \\ 0 \qquad\qquad\quad = \left(A_{21} + \overline{B}_{21}(n)\right) z_1(n) + \left(A_{22} + \overline{B}_{22}(n)\right) z_2(n) \end{cases} \tag{3.4}$$

If $A_{22} + \overline{B}_{22}(n)$ is invertible for all $n \in \mathbb{N}(n_0)$ then from the second equation of the system (3.4) we have

$$z_2(n) = -\left(A_{22} + \overline{B}_{22}(n)\right)^{-1}\left(A_{21} + \overline{B}_{21}(n)\right) z_1(n). \tag{3.5}$$

Substituting (3.5) into the first equation of the system (3.4) we obtain an ordinary difference equation

$$z_1(n+1) = \left[E_{11}^{-1}\left(A_{11} - A_{12}A_{22}^{-1}A_{21}\right) + E_{11}^{-1}R(n)\right] z_1(n), \tag{3.6}$$

where

$$\begin{aligned} R(n) = {} & \overline{B}_{11}(n) + \overline{B}_{12}(n)A_{22}^{-1}A_{21} - \overline{B}_{12}(n)\widetilde{B}_{22}(n)A_{21} \\ & - A_{12}\widetilde{B}_{22}(n)A_{21} + A_{12}A_{22}^{-1}\overline{B}_{21}(n) + \overline{B}_{12}(n)A_{22}^{-1}\overline{B}_{21}(n) \\ & - \overline{B}_{12}(n)\widetilde{B}_{22}(n)\overline{B}_{21}(n) - A_{12}\widetilde{B}_{22}(n)\overline{B}_{21}(n), \end{aligned}$$

with $\widetilde{B}_{22}(n) = A_{22}^{-1}\overline{B}_{22}(n)(A_{22} + \overline{B}_{22}(n))^{-1}$.

We make the following set of assumptions.

Assumption 2. $A_{22} + \overline{B}_{22}(n)$ is invertible , for all $n \in \mathbb{N}(n_0)$.

Assumption 3. There exists a constant $c > 0$ such that

$\left\|(A_{22} + \overline{B}_{22}(n))^{-1}(A_{21} + \overline{B}_{21}(n))\right\| < c$, for all $n \in \mathbb{N}(n_0)$.

Assumption 4.

$$\sum_{l=n_0}^{\infty} \left\|E_{11}^{-1}R(l)\right\| < \infty.$$

Assumption 5.

$\left\|E_{11}^{-1}R(l)\right\| \to 0$ as $l \to \infty$.

Theorem 3.2. Suppose that the implicit difference equation (2.3) has index-1, the finite eigenvalues of pencil $\{E, A\}$ have modulus less than or equal to one, and those of modulus one are semisimple. Let Assumptions 1, 2, 3 and 4 hold. Then, all solutions of (3.1) are bounded on $\mathbb{N}(n_0)$.

Proof. The properties of matrix pencil $\{E, A\}$ imply that all solutions of (3.1) are bounded on $\mathbb{N}(n_0)$. Under the Assumptions 1 and 2, every solution $u(n)$ of (3.1) is defined by $u(n) = Vz(n) = V\begin{pmatrix} z_1(n) \\ z_2(n) \end{pmatrix}$, where $z_1(n)$ is a solution of (3.6) and $z_2(n)$ is taken from (3.5). The Assumption 4 is satisfied, so applying the Theorem 2.3 we obtain $z_1(n)$ is bounded. Combining with Assumption 3, we imply that $z_2(n)$ is also bounded. Hence, the solution $u(n)$ of (3.1) is bounded. The proof is complete. □

Corollary 3.3. Suppose that the matrix pencil $\{E, A\}$ satisfies the conditions of Theorem (3.2). Let following conditions hold

i) $\sup\limits_{n\in\mathbb{N}(n_0)} \left\| E_{11}^{-1} F_{11}(n) \right\| < 1,$

ii) $\sup\limits_{n\in\mathbb{N}(n_0)} \left\| A_{22}^{-1}\left(B_{22}(n) - F_{21}(n)(E_{11} + F_{11}(n))^{-1}(A_{12} + B_{12}(n))\right) \right\| < 1,$

iii) For all $i, j \in \{1, 2\}$, $\sum\limits_{l=n_0}^{\infty} \left\| B_{ij}(l) \right\| < \infty,$

iv) For all $i \in \{1, 2\}$, $\sum\limits_{l=n_0}^{\infty} \left\| F_{i1}(l) \right\| < \infty,$

Then, all solutions of (3.1) are bounded on $\mathbb{N}(n_0)$.

Proof. Under conditions i)-iv), it is not difficult to verify that Assumptions 1-4 are satisfied. Thus, the conditions of Theorem 3.2 are fulfilled. Applying Theorem 3.2, the proof is complete. □

Theorem 3.4. Suppose that the implicit difference equation (2.3) has index-1, the finite eigenvalues of the matrix pencil $\{E, A\}$ are inside the unit disc. Let Assumptions 1, 2, 3, and 5 hold. Then, all solutions $u(n)$ of (3.1) tend to 0 as $n \to \infty$.

Proof. Because the finite eigenvalues of the matrix pencil $\{E, A\}$ are inside the unit disc, all solutions $x(n)$ of the difference equation (2.3) tend to zero as $n \to \infty$. Under the Assumptions 1 and 2, every solution $u(n)$ of (3.1) is defined by $u(n) = Vz(n) = V\begin{pmatrix} z_1(n) \\ z_2(n) \end{pmatrix}$, where $z_1(n)$ is a solution of (3.6) and $z_2(n)$ is taken from (3.5). The Assumption 5 is satisfied, so applying the Theorem 2.4 we obtain that $z_1(n)$ tend to zero as $n \to \infty$. Combining with Assumption 3, we imply that $z_2(n)$ is also tend to zero as $n \to \infty$. Hence, the solution $u(n)$ of (3.1) tend to zero as $n \to \infty$. The proof is complete. □

Corollary 3.5. Suppose that the matrix pencil $\{E, A\}$ satisfies the conditions of Theorem 3.4. Let following conditions hold

i) $\sup_{n\in\mathbb{N}(n_0)} \left\| E_{11}^{-1} F_{11}(n) \right\| < 1,$

ii) $\sup_{n\in\mathbb{N}(n_0)} \left\| A_{22}^{-1}\left(B_{22}(n) - F_{21}(n)(E_{11} + F_{11}(n))^{-1}(A_{12} + B_{12}(n)) \right) \right\| < 1,$

iii) For all $i, j \in \{1, 2\}$, $\left\| B_{ij}(n) \right\| \to 0$ as $n \to \infty$,

iv) For all $i \in \{1, 2\}$, $\left\| F_{i1}(n) \right\| \to 0$ as $n \to \infty$.

Then, all solutions $u(n)$ of (3.1) tend to 0 as $n \to \infty$.

Proof. Under conditions i)-iv), it is not difficult to verify that Assumptions 1-3 and 5 are satisfied. Thus, the conditions of Theorem 3.4 are fulfilled. Applying Theorem 3.4, the proof is complete. □

Example 3.6. Now we consider an example of (2.3), where

$$E = \begin{pmatrix} 3 & -1 \\ -6 & 2 \end{pmatrix}; \quad A = \begin{pmatrix} 5 & 5 \\ -5 & 5 \end{pmatrix}.$$

We see that $\sigma(E, A) = \{-1\}$. Hence, the equation (2.3) is stable.

Two perturbation matrices $F(n)$ and $B(n)$ of the perturbed equation (3.1)

$$B(n) = \begin{pmatrix} -\frac{15}{(n+1)^2} + \frac{3}{(n+2)^2} & \frac{5}{(n+1)^2} + \frac{9}{(n+2)^2} \\ -\frac{15}{(n+1)^2} + \frac{4}{(n+2)^2} & \frac{5}{(n+1)^2} + \frac{12}{(n+2)^2} \end{pmatrix}$$

$$F(n) = \begin{pmatrix} \frac{3}{(2n+3)^2} - \frac{6}{(n+3)^2} & -\frac{1}{(2n+3)^2} + \frac{2}{(n+3)^2} \\ -\frac{6}{(2n+3)^2} - \frac{3}{(n+3)^2} & \frac{2}{(2n+3)^2} + \frac{1}{(n+3)^2} \end{pmatrix}$$

Matrices U and V are chosen by

$$U = \begin{pmatrix} \frac{-1}{5} & \frac{2}{5} \\ \frac{2}{5} & \frac{1}{5} \end{pmatrix}; \quad V = \begin{pmatrix} \frac{-3}{10} & \frac{1}{10} \\ \frac{1}{10} & \frac{3}{10} \end{pmatrix};$$

We obtain

$$UAV = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad UEV = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix},$$

$$UF(n)V = \begin{pmatrix} \frac{1}{(2n+3)^2} & 0 \\ \frac{1}{(n+3)^2} & 0 \end{pmatrix}, \quad UB(n)V = \begin{pmatrix} \frac{1}{(n+1)^2} & \frac{1}{(n+2)^2} \\ \frac{3}{(n+1)^2} & \frac{2}{(n+2)^2} \end{pmatrix},$$

It is clear that conditions i)-iv) of Corollary 3.3 are satisfied. Applying Corollary 3.3, all solutions $u(k)$ of (3.1) are bounded.

Example 3.7. We consider an example where

$$E = \begin{pmatrix} 6 & 2 \\ -12 & -4 \end{pmatrix}; \quad A = \begin{pmatrix} 7 & -11 \\ -4 & -8 \end{pmatrix}$$

We see that $\sigma(E,A) = \left\{-\frac{1}{2}\right\}$. Hence, all solutions of the equation (2.3) tend to zero as $n \to \infty$. Two perturbation matrices $F(n)$ and $B(n)$ of the perturbed equation (3.1)

$$B(n) = \begin{pmatrix} \frac{42}{n+2} + \frac{6}{2n+2} & \frac{14}{n+2} - \frac{18}{2n+2} \\ \frac{6}{n+2} - \frac{2}{2n+2} & \frac{2}{n+2} + \frac{6}{2n+2} \end{pmatrix}$$

$$F(n) = \begin{pmatrix} \frac{6}{2n+3} + \frac{12}{n+3} & \frac{2}{2n+3} + \frac{4}{n+3} \\ -\frac{12}{2n+3} + \frac{6}{n+3} & \frac{-4}{2n+3} + \frac{2}{n+3} \end{pmatrix}$$

Choose

$$U = \begin{pmatrix} \frac{1}{10} & \frac{-1}{5} \\ \frac{1}{5} & \frac{1}{10} \end{pmatrix}; \quad V = \begin{pmatrix} \frac{3}{10} & \frac{1}{10} \\ \frac{1}{10} & \frac{-3}{10} \end{pmatrix};$$

We obtain

$$UAV = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{pmatrix}, \quad UEV = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix},$$

$$UF(n)V = \begin{pmatrix} \frac{1}{2n+3} & 0 \\ \frac{1}{n+3} & 0 \end{pmatrix}, \quad UB(n)V = \begin{pmatrix} \frac{1}{n+2} & \frac{1}{2n+2} \\ \frac{3}{n+2} & \frac{1}{2n+2} \end{pmatrix}$$

It is easy to verify that conditions i)-iv) of Corollary 3.5 are satisfied. Applying Corollary 3.5, all solutions $u(k)$ of (3.1) tend to 0 as $k \to \infty$.

4. Acknowledgment

The research presented here was done as a part of the project funded by Thang Long University according to decision 359/QDHT-DHTL.

References

[1] P. K. ANH, D. S. HOANG, Stability of a class of singular difference equations, Inter. J. Difference Equ., 1, 181-193 (2006).

[2] P. K. Anh, N. H. Du, L. C. Loi, Singular difference equations: an overview, Vietnam J. Math. 35, pp. 339- 372 (2007).

[3] P. K. Anh, N. H. Du, L. C. Loi, Connections between implicit difference equations and differential-algebraic equations, Acta Math. Vietnam. 29 (2004), pp. 23-39.

[4] P. K. Anh, H. T. N. Yen, On the solvability of initial-value problems for nonlinear implicit difference equations, Adv. Difference Eqns. 3 (2004), pp. 195-200.

[5] S. L. Campbell, Singular systems of differential equations I, Pitman Advanced Publishing Program, 1982.

[6] S. L. Campbell, Singular Systems of Differential Equations II, Pitman, London, 1982.

[7] L. Dai, Singular control systems, Lecture Notes in Control and Information Sciences 118, Springer-Verlag 1989.

[8] P. Kunkel and V. Mehrmann, Differential-Algebraic Equations, Analysis and Numerical Solution, European Math. Soc. Publ. House, 2006.

[9] L. C. Loi, Linear Implicit Nonautonomous Difference Equations, Ph.D. dissertation, Hanoi, Vietnam National Univ., 2004.

[10] R. Mӓrz, Numerical methods for differential-algebraic equations, Acta Numerica, 1 (1992), pp 141-198.

[11] B. Rodjanadid, N. V. Sanh, N. T. Ha, N. H. Du, Stability radii for implicit difference equations, Asian-European Journal of Mathematics, Vol. 2, No. 1 (2009) pp. 95-115.

[12] N. H. Du, V. H. Linh, On the robust stability of implicit linear systems containing a small parameter in the leading term, IMA J. Math. Control Inform, 23 (2006) 67-84.

[13] V. H. Linh, N. N. Tuan, Asymptotic integration of linear differential-algebraic equations, Electronic Journal of Qualitative Theory of Differential Equations, 2014, No. 12, 1-17.

[14] R.P. Agarwal, Difference Equations and Inequalities, Theory, Methods and Applications, vol. 228 of Monographs and Textbooks in Pure and Applied Mathematics, Marcel Dekker, New York, NY, USA, 2nd edition, 2000.

[15] T. Berger. Robustness of stability of time-varying index-1 DAEs. Preprint TUIlmenau, Germany, 2013.