ADAPTIVE NOISE CANCELLATION IMPLEMENTATION WITH A VARIABLE STEP-SIZE LMS ALGORITHM

Pham Tran Nhu
Institute of Information Technology
ptnhu@ioit.ac.vn

Hoang Manh Ha
College of BioMedical Instrumentations
1/89 Luong Dinh Cua Str – Phuong Mai Ward – Dong Da Dist – Ha Noi City
Viet Nam
hoangmanhha@yahoo.com

Abstract. An adaptive notch filter based on LMS Algorithm with variable step size parameter is designed for the elimination of power line interference in the recording of ECG signals

KEY WORDS

LMS: Least Mean Square.

1. Introduction

For emilinating the interference introduced by 50Hz power transmission lines in the recording of ECG and EEG signals, a Notch Filter was selected because it removes the power-line noise only, figure 1. In the case when the frequency of noise is not constant at exactly 50Hz the use of the Notch Filer

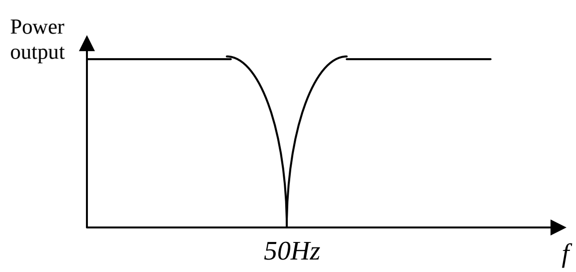

Figure 1: Central frequency and bandwidth of the ideal Fixed Notch Filter

implemented by Analog circuit (FNF) and designed as Band Pass Filter with bandwidth from 45Hz to 55Hz causes an information loss, figure 2.

To minimize information loss, ANF implemented with an adaptive algorithm was proposed in [1] and [2]. The algorithm adjusts a center frequency approach for

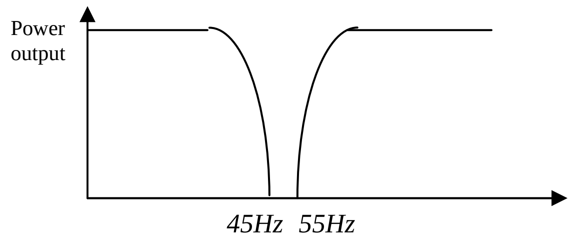

Figure 2: Central frequency and bandwidth of the reality Fixed Notch Filter

noise's frequency after converging time. Furthermore, in ANF the bandwidth depends on a step size parameter μ of the bandwidth $2\mu C^2$. If step size is chosen in range $0<\mu<\frac{1}{\lambda_{max}}$ then algorithm is stable (see [2]) and the band width is enough narrow for minimization of information loss. After some experimentations of ANF we recognized that, at some places in Vietnam, quality of electricity supplies is so bad due to use of small dynamo and changing of noise's frequency is larger and faster. This situation requires an adaptive algorithm with stability and shorter time in converging. Our solution was proposed for this problem. Further more, value of μ is small enough to keep a usefull information in ECG signals

2. Description

2.1 An Adaptive Notch Filter

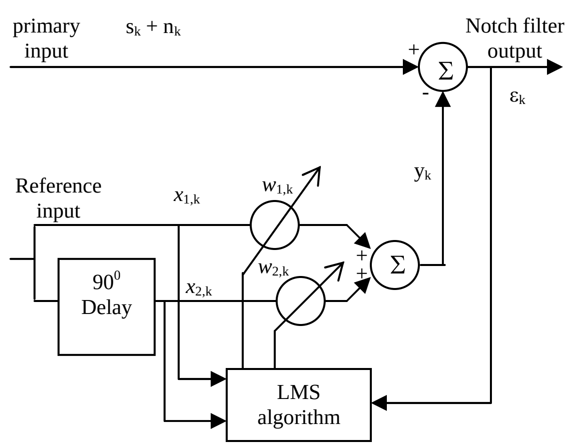

Figure. 3: Model of noise canceling used a adaptive filter

Where

s_k: Clear BioMedical Signal, at k

n_k: Additive interferences, at k

$s_k + n_k$: receiving signal, content Clear BioMedical Signal and interferences, at k

$x_{1,k}$: Receiving interference from Reference input, at k

$x_{2,k}$: Delay$90^0[x_{1,k}]$

$x_{1,k}$ and $x_{2,k}$ are described below

$$x_{1,k} = C\cos(k\omega_0 + \phi)$$

$$x_{2,k} = C\sin(k\omega_0 + \phi)$$

$$y_k = x_{1,k}w_{1,k} + x_{2,k}w_{2,k}$$

$$\varepsilon_k = s_k + n_k - y_k$$

$w_{1,k}$ and $w_{2,k}$: Weights of Adaptive Filter, their updating as below (see[2])

$$w_{1,k+1} = w_{1,k} + 2\mu\varepsilon_k x_{1,k}$$
$$w_{2,k+1} = w_{2,k} + 2\mu\varepsilon_k x_{2,k}$$

μ:: Step size parameter

if $0 < \mu < \dfrac{1}{\lambda_{\max}}$ then LMS algorithm is stable

$BandWidth = 2\mu C^2\ rad$ (see[2])

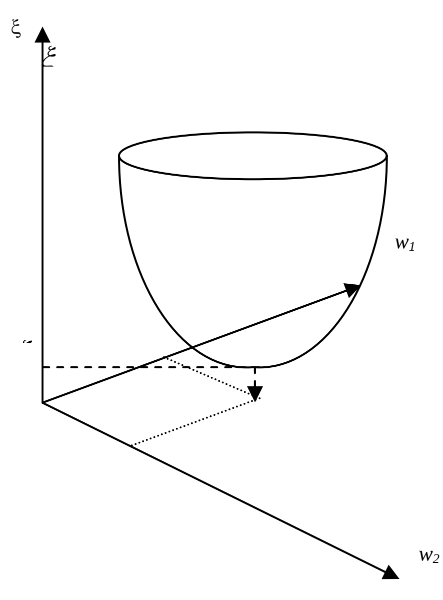

Figure 4: Quadric Performance Surface

It mean central frequency of adaptive notch filter reached to ω_0 (frequency of interferences), therefore only frequency of interferences is cancel,

A adaptation process corresponds with a process of approximation to minimum point in performance surface

2.2 An Adaptive Notch Filter with Variable step Size

In an adaptive notch filter, as presented above, stability condition is a slow adaptation (small value of μC^2) (see [2]). This is not suitable for complex noise environment in which double faster convergnce and stability are required. Recenly, there are some approach to solve this problem:

- Improving performance of algorithm by parallel programming. This requires conditions about hardware and programmer's experience
- Changing Filter's order.
- Replace μ constant with function (see[1]), In [1] authors use the following formula to describe a step size change

$$\mu(k+1)=\alpha\mu(k)+\gamma e^2(k) \quad (12)$$

where:

α: is a forgetting factor with its value in range [0,1]

γ: is step size parameter for the adaptation of μ

In general cases, *e(n)* is an error at filter's output, so it is smooth. But to denoise by using a adaptive filter model, *e(n)* is filtered signal. In the case of ECG signals, R peaks change suddenlly. For this case algorithm is converged but it is going to leave stable status immediately due to value of *e(n)* [2].

To find a more suitable solution we recognised laws of gradient's distribution on quadric performance surface as described bellow

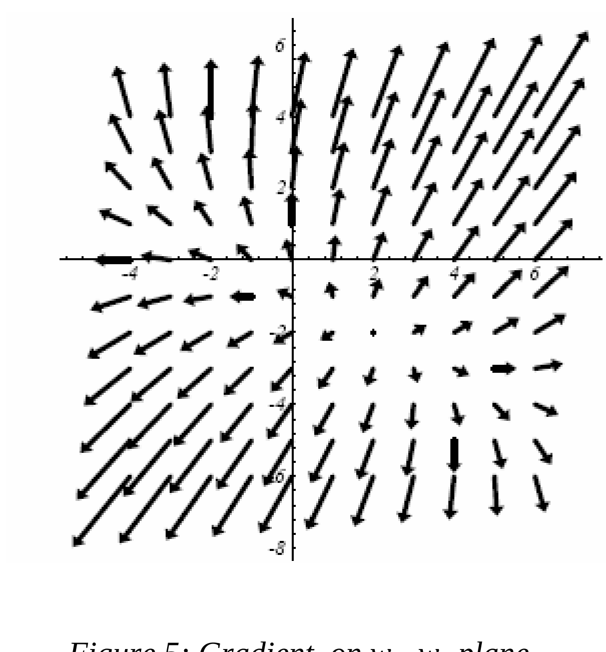

Figure 5: Gradient on w_1, w_2 plane

- Gradient's direction orthogonal to contour line of quadric performance surface (see [2] & [3])

- Gradient's projection on (w_1, w_2)-plane has magnitude depending on its distance to the optimal point. Its magnitude becomes smaller if the point under consideration is closer to the optimal point

Hence,

$$\begin{aligned} \mu_{1,k+1} &= \mu_{1,k} + x_{1k}\left(\varepsilon_k - \varepsilon_{k-1}\right) \\ \mu_{2,k+1} &= \mu_{2,k} + x_{2k}\left(\varepsilon_k - \varepsilon_{k-1}\right) \end{aligned} \quad (13)$$

Where

$\mu_{1,k}$: a step size for adjust w_1 at k

$\mu_{2,k}$: a step size for adjust w_2 at k

$x_{1,k}$, $x_{2,k}$: reference inputs at k

ε_k: Output of Noise canceler at k

Formular (13) allow a value of μ is adjusted optimally in lowest complexity of computing

2.3 Experiments and results

LMS algorithm is implemented by equations (3), (4), and (13)

For an evaluation, we compute a Mean Squared Error (MSE) use equation

$$MSE(nT) = \frac{\sqrt{\sum_{i=1}^{T}(e(i)-s(i))^2}}{T} \quad (14)$$

Where

e(i): Notch Filter's output

s(i): Without Noise ECG Signals

n=1,2 ... (Length of signal/10)

We choose T=10 in experiments bellow

Noise's frequency is shifted every 3 seconds

We use Matlab for experiments

Experiment 1:

To demonstrate a relation between stability and step size μ we consider $\mu=3$ *and* $\mu=5$

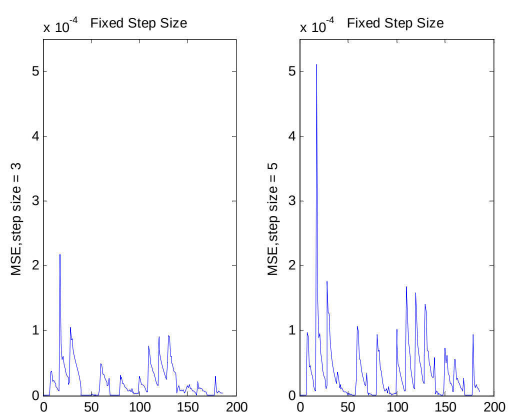

Figure 6: Adaptive Notch Filter with a Fixed step size

At figure 6, case μ=3 (left) is more stabilizable than case μ=5 (right). This also will be shown by Figure 7 and 8 bellow

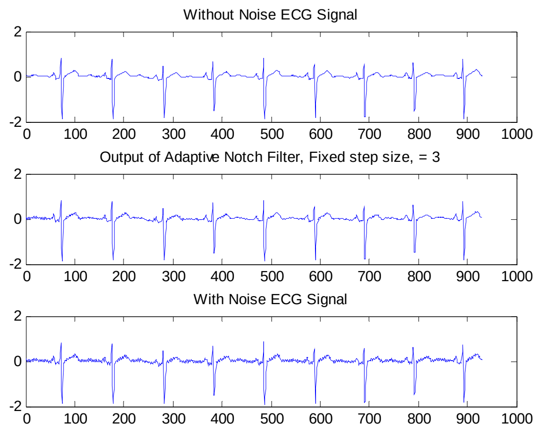

Figure 7: Output of Adaptive Notch Filter. Fixed Step Size μ=3

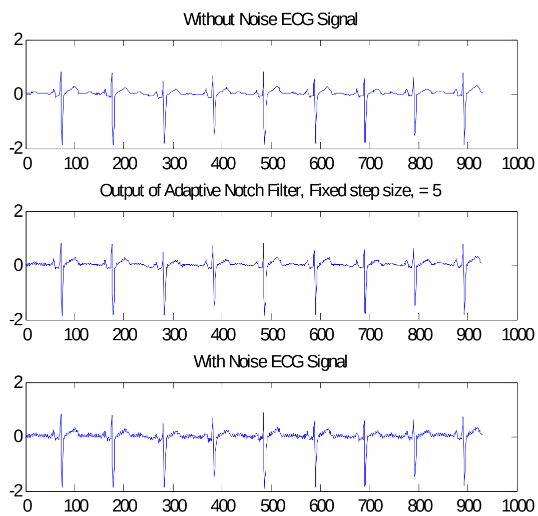

Figure 8: Output of Adaptive Notch Filter. Fixed Step Size μ=5

In case μ=3 (figure 7), Notch filter's output progress to better step by step, although with low rate. In case μ=5 (figure 8) there are a lot of noise in Notch filter's output

Experiment 2

In this experiment, comparision of Adaptive Notch Filters, Fixed Step Size and Variable Step Size.

At begin of Converging progress, $\mu=3$ for both of filters (Variable Step Size Adaptive Notch Filter and Fixed Step Size Adaptive Filter)

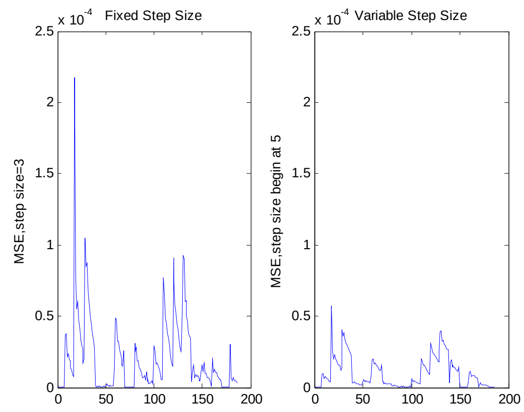

Figure9: MSE in case initializing μ =3

At figure 9, case of varriable step size (right) is more stabilizable than case of fixed step size (left) In the case when we use variable step size, the MSE is smaller and reaches the zeros value faster than the one with fixed step size. Moreover, MSE doesn't leave zeros value after convergence. This also will be shown by Figure 11

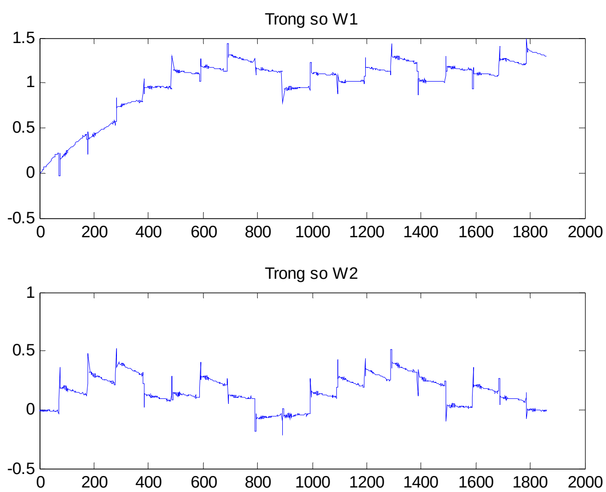

Figure 10: change of weights, W1 and W2

Figuge 10 describe a changes of w_1 and w_2 weights (see Model of noise canceling used a adaptive filter in Figure. 3)

w1 (upper) seem converged around 1, w1 (under) seem converged around 0

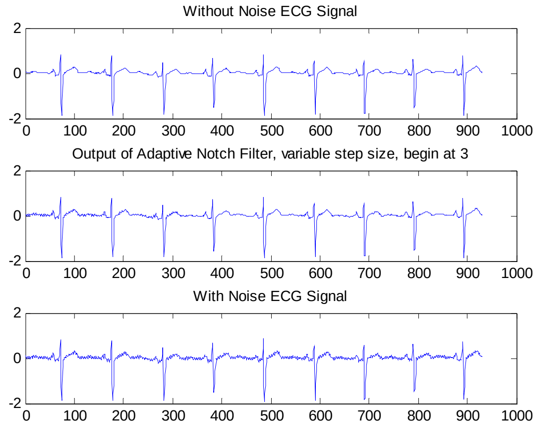

Figure 11: pure ecg signal, after denoise signal, noisy ecg signal

In figure 11, pure ecg signal is located at top, noisy ecg signal is located at bottom, after denoise ecg signal located at medium. Easy to recognise that Noise canceller with variable step size is more stabilizable and has a faster convergence than the one with fixed step size (compare figure 11 with figure 7).

Experiment 3:

In this experiment, comparision of Adaptive Notch Filters, Fixed Step Size and Variable Step Size is repeated, but at begin of Converging progress, μ=5 is chosen for both of filters (Variable Step Size Adaptive Notch Filter and Fixed Step Size Adaptive Filter)

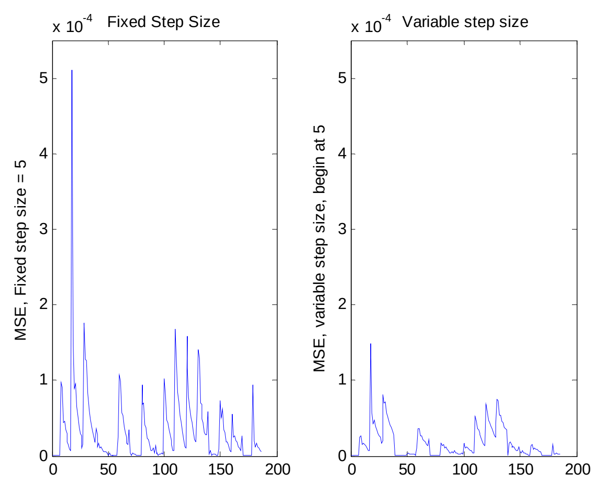

Figure 13: MSE in case initializing μ =5

At figure 13, case of varriable step size (right) is more stabilizable than case of fixed step size (left) In the case when we use variable step size, the MSE is smaller and reaches the zeros value faster than the one with fixed step size. This also will be shown by Figure 15

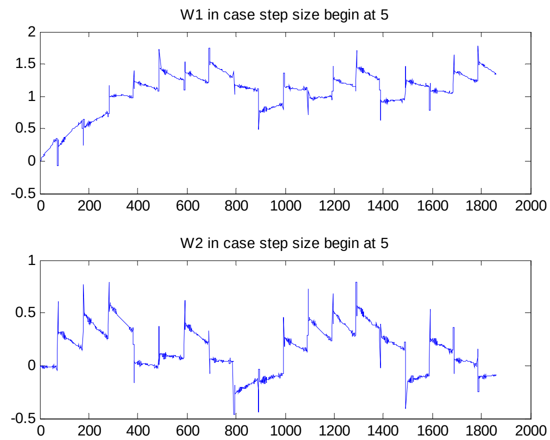

Figure 14: change of weights, W1 and W2

Figuge 14 describe a changes of w1 (upper), w1 (under) both seem converged but not stronger than case of experiment 2

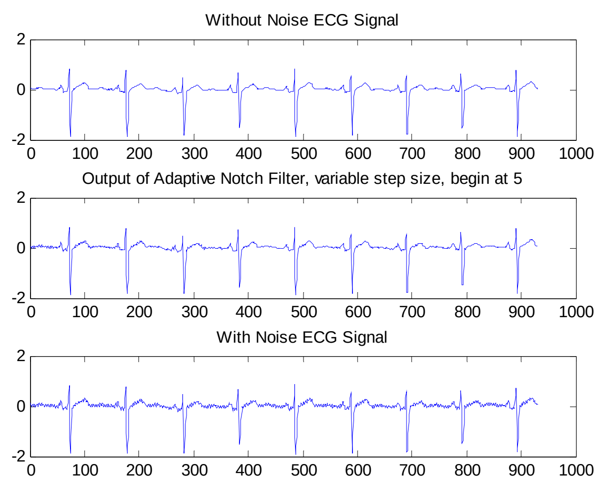

Figure 15: pure ecg signal, after denoise signal, noisy ecg signal

In figure 15, pure ecg signal is located at top, noisy ecg signal is located at bottom, after denoise ecg signal located at medium. Easy to recognise that Noise canceller with variable step size is more stabilizable and has a faster convergence than the one with fixed step size (compare figure 11 with figure 8).

3. Conclusion

From the above mentioned experiment results, Adaptive Notch Filter with variable step size have faster convergence and more stabilizable than Adaptive Notch Filter with Fixed step size. For Adaptive Notch Filter with Fixed step size, if μ does not belong to $\left[0, \frac{1}{\lambda_{\max}}\right]$ then algorithm convergence will not be able. Experiment 1 demonstrated this affirmation by choosing μ=3 (stability) and μ=5 (not stability). The experiment 2 and experiment 3 have shown that by using LMS algorithm with variable step size, we alway get stability. In the cases of μ getting 3 and 5, the algorithm is stable.

4. References

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[3] Pham Tran Nhu and Hoang Manh Ha, Choise of Markov step-size for least mean-square (LMS) weights convergence, Scientific Conference '06 of Institute of Information Technology, Hanoi 27-28/12/2006.