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THE NONLINEAR ABSORPTION COEFFICIENT OF A STRONG ELECTROMAGNETIC WAVE BY CONFINED ELECTRONS IN QUANTUM WELLS UNDER THE INFLUENCES OF CONFINED PHONONS

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Abstract—The nonlinear absorption coefficient (NAC) of a strong electromagnetic wave (EMW) by confined electrons in quantum wells under the influences of confined phonons is theoretically studied by using the quantum transport equation for electrons. In comparison with the case of unconfined phonons, the dependence of the NAC on the energy ($\hbar\Omega$), the amplitude (E_o) of external strong EMW, the width of quantum wells (L) and the temperature (T) of the system in both cases of confined and unconfined phonons is obtained. Two limited cases for the absorption: close to the absorption threshold ($|k\hbar\Omega - \hbar\omega_0| \ll \bar{\varepsilon}$) and far away from the absorption threshold ($|k\hbar\Omega - \hbar\omega_o| \gg \bar{\varepsilon}$) ($k = 0, \pm1, \pm2, \ldots, \omega_o$ and $\bar{\varepsilon}$ are the frequency of optical phonon and the average energy of electron, respectively) are considered. The formula of the NAC contains the quantum number m characterizing confined phonons and is easy to come back to the case of unconfined phonons and linear absorption. The analytic expressions are numerically evaluated, plotted and discussed for a specific case of the GaAs/GaAsAl quantum well. Results show that there are more resonant peaks of the NAC which appear in the case of confined phonons when $\Omega > \omega_0$ than in that of unconfined phonons. The spectrums of the NAC are very different from the linear absorption and strongly depend on m.

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1. INTRODUCTION

Recently, there are more and more interest in studying and discovering the behavior of low-dimensional system, in particular two-dimensional systems, such as semiconductor superlattices, quantum wells and doped superlattices (DSLs). The confinement of electrons in low-dimensional systems considerably enhances the electron mobility and leads to unusual behaviors under external stimuli. Many attempts have been conducted dealing with these behaviors, for examples, electron-phonon interaction effects on two-dimensional electron gases (graphene, surfaces, quantum wells) [1, 8, 10]. The dc electrical conductivity [2, 3], electronic structure [18], wavefunction distribution [19] and electron subband [20] in quantum wells have been calculated and analyzed. The problems of the absorption coefficient for a weak EMW in quantum wells [4], DSLs [5] and quantum wires [15] have also been investigated by using Kubo-Mori method. The experimental and theoretical investigations of the linear and nonlinear optical properties in semiconductor quantum wells [6] which including the effects of electrostatic fields, extrinsic carriers and real or virtual photocarriers were reviewed. The absorption coefficients for the intersubband transitions with influences of the linear and nonlinear optical properties in multiple quantum wells accounted fully for the experimental results [9] and were calculated by using a combination of quantum genetic algorithm (QGA) and hartree-fock roothan (HFR) method in quantum dots [12]. The linear and nonlinear optical absorption coefficients in quantum dots were investigated by using QGA, HFR and the potential morphing method in the effective mass approximation [11, 13]. The nonlinear absorption of a strong EMW by confined electrons in rectangular quantum wires [14] have been studied by using the quantum transport equation for electrons.

However, However, the nonlinear absorption problem of an EMW which has strong intensity and high frequency with case of confined phonons is stills open to study. So in this paper, we study the NAC of a strong EMW by confined electrons in quantum wells under the influences of confined phonons. Then, we estimate numerical values for a specific AlAs/GaAs/AlAs quantum well to clarify our results.

2. NONLINEAR ABSORPTION COEFFICIENT IN CASE OF CONFINED PHONONS

It is well-known that the motion of an electron is confined in each layer of the DSL, and its energy spectrum is quantized into discrete levels. In this article, we assume that the quantization direction is in

z direction and only consider intersubband transitions ($n \neq n'$) and intrasubband transitions ($n = n'$). The Hamiltonian of the confined electron-confined optical phonon system in quantum wells in the second quantization representation can be written as:

$$H = H_o + U \tag{1}$$

$$H_o = \sum_{\mathbf{k}_\perp,n} \varepsilon_n \left(\mathbf{k}_\perp - \frac{e}{\hbar c}\mathbf{A}(t)\right) a^+_{\mathbf{k}_\perp,n} a_{\mathbf{k}_\perp,n} + \sum_{\mathbf{q}_\perp,m} \hbar\omega_o b^+_{\mathbf{q}_\perp,m} b_{\mathbf{q}_\perp,m} \tag{2}$$

$$U = \sum_{\mathbf{k}_\perp,n,n'} \sum_{\mathbf{q}_\perp,m} C_{\mathbf{q}_\perp,m} I^m_{nn'} a^+_{\mathbf{k}_\perp+\mathbf{q}_\perp,n'} a_{\mathbf{k}_\perp,n} \left(b^+_{-\mathbf{q}_\perp,m} + b_{\mathbf{q}_\perp,m}\right) \tag{3}$$

where H_o is the non-interaction Hamiltonian of the confined electron-confined optical phonon system, and n ($n = 1, 2, 3, \ldots$) denotes the quantization of the energy spectrum in the z direction. $(\mathbf{k}_\perp, n)$ and $(\mathbf{k}_\perp + \mathbf{q}_\perp, n')$ are electron states before and after scattering, and $(\mathbf{k}_\perp, \mathbf{q}_\perp)$ is the in plane $(x,\, y)$ wave vector of the electron (phonon). $a^+_{\mathbf{k}_\perp,n}, a_{\mathbf{k}_\perp,n}$ $(b^+_{\mathbf{q}_\perp,m}, b_{\mathbf{q}_\perp,m})$ are the creation and the annihilation operators of the electron (phonon), respectively, and $\mathbf{A}(t)$ is the vector potential of an external EMW $\mathbf{A}(t) = \frac{e}{\Omega}\mathbf{E}_o \sin(\Omega t)$. $\hbar\omega_o$ is the energy of an optical phonon. The electron energy $\varepsilon_{\mathbf{k}_\perp,n}$ in quantum wells takes the simple form [7]:

$$\varepsilon_{\mathbf{k}_\perp,n} = \frac{\pi^2\hbar^2}{2m_e L^2} n^2 + \frac{\hbar^2}{2m_e}\mathbf{k}^2_\perp \tag{4}$$

Here, m_e and e are the effective mass and the charge of the electron, respectively. L is the width of quantum wells, and $C_{\mathbf{q}_\perp,m}$ is the electron-phonon interaction potential. In the case of the confined electron-confined optical phonon interaction, we assume that the quantization direction is in z direction, and $C_{\mathbf{q}_\perp,m}$ is:

$$|C_{\mathbf{q}_\perp,m}|^2 = \frac{2\pi e^2 \hbar\omega_o}{\varepsilon_o V}\left(\frac{1}{\chi_\infty} - \frac{1}{\chi_o}\right)\frac{1}{\mathbf{q}^2_\perp + \left(\frac{m\pi}{L}\right)^2} \tag{5}$$

where V and ε_o are the normalization volume and the electronic constant (often $V = 1$), and $m = 1, 2, \ldots$, is the quantum number m characterizing confined phonons. χ_o and χ_∞ are the static and high-frequency dielectric constant, respectively. The electron form factor in case of unconfined phonons is written as [1]:

$$I^m_{nn'} = \frac{2}{L}\int_0^L \left[\eta(m)\cos\frac{m\pi z}{L} + \eta(m+1)\sin\frac{m\pi z}{L}\right]\sin\frac{n'\pi z}{L}\sin\frac{n\pi z}{L}dz \tag{6}$$

With $\eta(m) = 1$ if m is even number and $\eta(m) = 0$ if m is odd number.

In order to establish the quantum kinetic equations for the electrons in quantum wells in the case of confined phonons, we use general quantum equation for the particle number operator (or electron distribution function) $n_{\mathbf{k}_\perp,n} = \left\langle a^+_{\mathbf{k}_\perp,n} a_{\mathbf{k}_\perp,n} \right\rangle_t$:

$$i\hbar \frac{\partial n_{\mathbf{k}_\perp,n}}{\partial t} = \left\langle a^+_{\mathbf{k}_\perp,n} a_{\mathbf{k}_\perp,n}, H \right\rangle_t \tag{7}$$

where $\langle \psi \rangle_t$ is the statistical average value at the moment t and $\langle \psi \rangle_t = Tr(\hat{W} \overset{\wedge}{\psi})$ ($\hat{W}$ being the density matrix operator).

Because the motion of electrons is confined along z direction in quantum wells, we only consider the in plane (x, y) current density vector of electrons so the carrier current density formula in quantum wells takes the form:

$$\mathbf{j}_\perp(t) = \frac{e\hbar}{m_e} \sum_{\mathbf{k}_\perp,n} \left(\mathbf{k}_\perp - \frac{e}{\hbar c} \mathbf{A}(t) \right) n_{\mathbf{k}_\perp,n} \tag{8}$$

The NAC of a strong EMW by confined electrons in the two-dimensional systems takes the simple form:

$$\alpha = \frac{8\pi}{c\sqrt{\chi_\infty} E_o^2} \left\langle \mathbf{j}_\perp(t) \mathbf{E}_o \sin \Omega t \right\rangle_t \tag{9}$$

Starting from Hamiltonian (1, 2, 3) and realizing operator algebraic calculations, we obtain the quantum kinetic equation for electrons in quantum wells. After using the first order tautology approximation method to solve this equation, the expression of electron distribution function can be written as:

$$\begin{aligned}
n_{\mathbf{k}_\perp,n}(t) \;=\; & -\frac{1}{\hbar^2} \sum_{\mathbf{q}_\perp,m,n'} \left| C_{\mathbf{q}_\perp,m} \right|^2 \left| I^m_{n,n'} \right|^2 \\
& \sum_{k,l=-\infty}^{+\infty} \frac{1}{l\Omega} J_k\left(\frac{\lambda}{\Omega}\right) J_{k+l}\left(\frac{\lambda}{\Omega}\right) \exp(-il\Omega t) \\
& \times \left\{ \frac{\bar{n}_{\mathbf{k}_\perp-\mathbf{q}_\perp,n'} N_{\mathbf{q}_\perp,m} - \bar{n}_{\mathbf{k}_\perp,n}(1+N_{\mathbf{q}_\perp,m})}{\varepsilon_n(\mathbf{k}_\perp) - \varepsilon_{n'}(\mathbf{k}_\perp - \mathbf{q}_\perp) - \hbar\omega_o - k\hbar\Omega + i\delta} \right. \\
& + \frac{\bar{n}_{\mathbf{k}_\perp-\mathbf{q}_\perp,n'}(1+N_{\mathbf{q}_\perp,m}) - \bar{n}_{\mathbf{k}_\perp,n} N_{\mathbf{q}_\perp,m}}{\varepsilon_n(\mathbf{k}_\perp) - \varepsilon_{n'}(\mathbf{k}_\perp - \mathbf{q}_\perp) - \hbar\omega_o - k\hbar\Omega + i\delta} \\
& - \frac{\bar{n}_{\mathbf{k}_\perp,n} N_{\mathbf{q}_\perp,m} - \bar{n}_{\mathbf{k}_\perp+\mathbf{q}_\perp,n'}(1+N_{\mathbf{q}_\perp,m})}{\varepsilon_{n'}(\mathbf{k}_\perp) - \varepsilon_{n'}(\mathbf{k}_\perp - \mathbf{q}_\perp) - \hbar\omega_o - k\hbar\Omega + i\hbar\delta} \\
& \left. - \frac{\bar{n}_{\mathbf{k}_\perp,n}(1+N_{\mathbf{q}_\perp,m}) - \bar{n}_{\mathbf{k}_\perp+\mathbf{q}_\perp,n'} N_{\mathbf{q}_\perp,m}}{\varepsilon_{n'}(\mathbf{k}_\perp + \mathbf{q}_\perp) - \varepsilon_n(\mathbf{k}_\perp) + \hbar\omega_o - k\hbar\Omega + i\hbar\delta} \right\}
\end{aligned} \tag{10}$$

where $\bar{n}_{\mathbf{k}_\perp,n}$ is the time-independent component of the electron distribution function; $J_k(x)$ is the Bessel function; $N_{\mathbf{q}_\perp,m}$, which comply with Bose-Einstein statistics, is the time-independent component of the phonon distribution function [16]. In the case of the confined electron-confined optical phonon interaction, the phonon distribution function $N_{\mathbf{q}_\perp,m}$ can be written as [17]:

$$N_{\mathbf{q}_\perp,m} = \frac{1}{e^{\frac{\hbar\omega_o}{k_B T}} - 1} \tag{11}$$

By using Eq. (10), the electron-optical phonon interaction factor $C_{\mathbf{q}_\perp,m}$ in Eq. (5) and the Bessel function, from the expression of current density vector in Eq. (8) and the relation between the NAC of a strong EMW with $\mathbf{j}_\perp(t)$ in Eq. (9), we established the NAC of a strong EMW in quantum wells:

$$\alpha = \frac{16\pi^3 e^2 \Omega k_B T}{\varepsilon_o c\sqrt{\chi_\infty} E_o^2}\left(\frac{1}{\chi_\infty} - \frac{1}{\chi_o}\right)\sum_{m,n,n'}\sum_{\mathbf{k}_\perp,\mathbf{q}_\perp}\sum_{k=1}^{\infty}|I_{nn'}^m|^2(\bar{n}_{\mathbf{k}_\perp,n} - \bar{n}_{\mathbf{k}_\perp+\mathbf{q}_\perp,n'})$$
$$\times\frac{kJ_k^2\left(\frac{\lambda}{\Omega}\right)}{\mathbf{q}_\perp^2 + (m\pi/L)^2}\delta(\varepsilon_{\mathbf{k}_\perp+\mathbf{q}_\perp,n'} - \varepsilon_{\mathbf{k}_\perp,n} + \hbar\omega_o - k\hbar\Omega),\ \text{with}\ \lambda = \frac{e\mathbf{E}_o\mathbf{q}_\perp}{m_e\Omega} \tag{12}$$

Equation (12) is the general expression for the NAC of a strong EMW in quantum wells. In this paper, we will consider two limited cases for the absorption, close to the absorption threshold and far away from absorption threshold, to find out the explicit formula for the NAC.

2.1. The Absorption Far away from Threshold

In this case, for the absorption of a strong EMW in a quantum well the condition $|k\hbar\Omega - \hbar\omega_o| \gg \bar{\varepsilon}$ must be satisfied. Here, $\bar{\varepsilon}$ is the average energy of an electron in quantum wells. Finally, we have the explicit formula for the NAC of a strong EMW in quantum wells for the case of the absorption far away from its threshold, which is written as:

$$\alpha = \frac{8\pi^2 e^4 k_B T n_o^*}{c\varepsilon_o\sqrt{\chi_\infty}\hbar^2 L m_e \Omega^3}\left(\frac{1}{\chi_\infty} - \frac{1}{\chi_o}\right)\times\left[1 - \exp\left[\frac{\hbar}{k_B T}(\omega_o - \Omega)\right]\right]$$
$$\times\sum_{m,n,n'}|I_{nn'}^m|^2\times\left\{1 - \frac{3}{8}\left(\frac{eE_o}{2m_e\Omega^2}\right)^2\lambda_o\right\}\lambda_o^{3/2}\left[(m\pi/L)^2 - \lambda_o\right]^{-1} \tag{13}$$

With: $\lambda_o = \frac{2m_e}{\hbar^2}\left(\left(n'^2 - n^2\right)\varepsilon_o + \hbar\omega_o - \hbar\Omega\right)$, $n_o^* = \frac{n_o e^{3/2}\pi^{3/2}\hbar^3}{V m_e^{3/2}(k_B T)^{3/2}}$ (n_o is the electron density in quantum wells), and k_B is Boltzmann constant.

When quantum number m characterizing confined phonons reaches zero, the expression of the NAC for the case of absorption far away from its threshold in quantum wells without influences of confined phonons can be written as:

$$\alpha = \frac{4\pi^2 e^4 k_B T n_o^*}{c\varepsilon_o\sqrt{\chi_\infty}\hbar^2 L m_e \Omega^3}\left(\frac{1}{\chi_\infty}-\frac{1}{\chi_o}\right)\times\sum_{n,n'}\left[\frac{2m_e}{\hbar}(\Omega-\omega_o)+\frac{\pi^2\left(n^2-n'^2\right)}{L^2}\right]^{\frac{1}{2}}$$
$$\times\left\{1+\frac{3}{8}\left(\frac{eE_o}{2m_e\Omega^2}\right)^2\left[\frac{2m_e}{\hbar}(\Omega-\omega_o)+\frac{\pi^2\left(n^2-n'^2\right)}{L^2}\right]\right\}$$
$$\times\left[1-\exp\left[\frac{\hbar}{k_B T}(\omega_o-\Omega)\right]\right] \tag{14}$$

2.2. The Absorption Close to the Threshold

In this case, the condition $|k\hbar\Omega - \hbar\omega_o| \ll \bar{\varepsilon}$ is needed. Therefore, we cannot ignore the presence of the vector $\mathbf{k}_\perp$ in the formula of δ function. This also means that the calculation depends on the electron distribution function $n_{n,\mathbf{k}_\perp}$. Finally, the expression for the NAC of a strong EMW in quantum wells in the case of absorption close to its threshold is obtained:

$$\alpha = \frac{e^4 n_o^*(k_B T)^2}{c\varepsilon_o\sqrt{\chi_\infty}\Omega^3\hbar^4 L}\left(\frac{1}{\chi_\infty}-\frac{1}{\chi_o}\right)$$
$$\times\left\{1-\exp\left[\frac{\hbar}{k_B T}(\omega_o-\Omega)\right]\right\}\sum_{mnn'}|I_{nn'}^m|^2\exp\left(-\frac{\pi^2 n^2}{2m_e k_B T L^2}\right)$$
$$\times\exp\left[-\frac{\hbar^2}{4m_e k_B T}(\lambda_o+|\lambda_o|)\right]\left[1+\frac{3}{8}\frac{e^2E_o^2}{\hbar^2 m_e\Omega^4}\left(1+\frac{\hbar^2}{4m_e k_B T}|\lambda_o|\right)\right] \tag{15}$$

When quantum number m characterizing confined phonons reaches zero, the expression of the NAC for the case of absorption far away from its threshold in quantum wells without influences of confined phonons can be written as:

$$\alpha = \frac{e^4 n_o^*(k_B T)^2}{2c\varepsilon_o\sqrt{\chi_\infty}\Omega^3\hbar^4 L}\left(\frac{1}{\chi_\infty}-\frac{1}{\chi_o}\right)$$
$$\times\left\{\exp\left[\frac{\hbar}{k_B T}(\Omega-\omega_o)\right]-1\right\}\sum_{nn'}\exp\left(-\frac{\pi^2 n'^2}{2m_e k_B T L^2}\right)$$
$$\times\left\{1+\frac{3e^2 k_B T}{8m_e\hbar^2\Omega^4}E_o^2\left[1+\frac{1}{2k_B T}\times\left(\frac{\pi^2\hbar^2\left(n'^2-n^2\right)}{2m_e L^2}+\hbar(\omega_o-\Omega)\right)\right]\right\} \tag{16}$$

In Eq. (16), we can see that the formula of the NAC is easy to come back to the case of linear absorption when the intensity (E_o)

of external EMW reaches zero which was calculated by Kubo-Mori method [4].

3. NUMERICAL RESULTS AND DISCUSSION

In order to clarify the mechanism for the NAC of a strong EMW in a quantum well with the case of confined, in this section, we will evaluate, plot and discuss the expression of the NAC for a specific quantum well: AlAs/GaAs/AlAs. We use some results for linear absorption in [4] to make the comparison. The parameters used in the calculations are as follows [4, 5]: $\chi_o = 12.9$, $\chi_\infty = 10.9$, $n_o = 10^{23}$, $L = 100A^0$, $m_e = 0.067m_0, m_0$ being the mass of free electron, $\hbar\omega_o = 36.25\,\mathrm{meV}$ and $\Omega = 2.10^{14}\,\mathrm{s}^{-1}$.

3.1. The Absorption Far away from Its Threshold

Figures 1 and 2 show the NAC of a strong EMW as a function of the amplitude E_0 of a strong EMW and the temperature T of the system in a quantum well for the case of the absorption far away from its threshold. The curve of the NAC increases following the amplitude E_0 rather fast, and when the temperature T of the system rises up, it is quite linearly dependent on T. The spectrums of the NAC are much different from linear absorption coefficient [4] but quite similar to the NAC of a strong EMW in rectangular quantum wires [14]. The values of NAC increase following the temperature T much more strongly than in case of linear absorption.

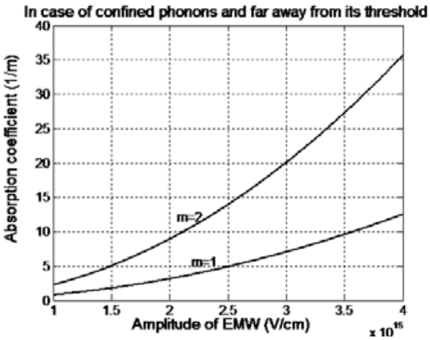

Figure 1. The dependence of α on E_o in case of confined phonons.

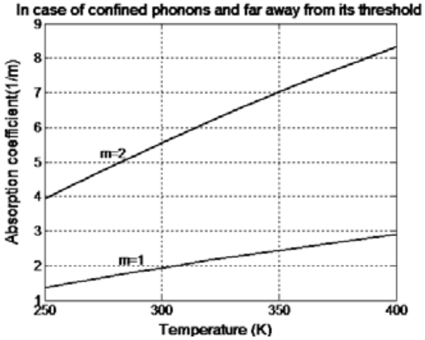

Figure 2. The dependence of α on T in case of confined phonons.

3.2. The Absorption Close to the Threshold

In this case, the dependence of the NAC on other parameters is quite similar with case of the absorption far away from its threshold. But, the values of the NAC are much greater than the above case. Also, it is seen that the absorption coefficient depends on the energy of EMW $\hbar\Omega$, and the width of quantum wells L is much stronger than in the case of linear absorption [4]. Especially, Figure 3 shows that there are clearly two resonant peaks of the NAC which is similar to the total optical absorption coefficient in quantum dots in [11, 13]. The first resonant peak which appears at $\Omega = \omega_o$ is similar to the case of unconfined phonons (in figure 5), the linear absorption [4] and the NAC of a strong EMW in rectangular quantum wires [14]. The second one which appears when $\Omega > \omega_o$ is higher than the first one. In Figure 4, each curve has one maximum peak when the width of quantum wells L varies from 20 nm to 40 nm. When we consider the case $E_o = 0$ in Eq. (16), the nonlinear results will turn back to linear results which were calculated by using the Kubo-Mori method [4].

Figures 1–4 show that the NAC depends very strongly on quantum number m characterizing confined phonons. The NAC gets stronger when the confinement of phonons increases. In Figure 5, when the quantum number m characterizing confined phonons reaches zero in Eq. (16), we will get the results of the NAC in case of unconfined phonons. Figure 5 shows that the resonant peak of the absorption coefficient in case of nonlinear absorption appears more clearly and higher than in case of linear absorption [4].

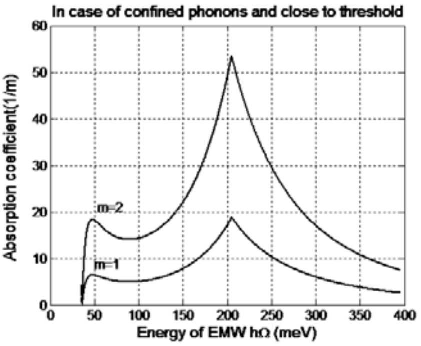

Figure 3. The dependence of α on $\hbar\Omega$ in case of confined phonons.

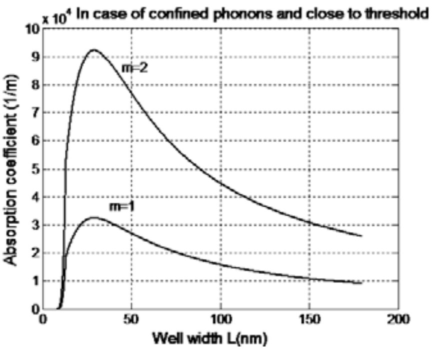

Figure 4. The dependence of α on L in case of confined phonons.

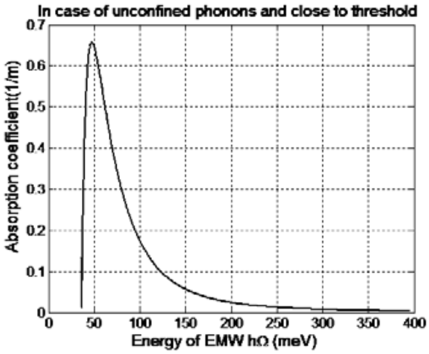

Figure 5. The dependence of α on $\hbar\Omega$ in case of unconfined phonons.

4. CONCLUSION

In this paper, we have theoretically studied the nonlinear absorption of a strong EMW by confined electrons in quantum wells under the influences of confined phonons. We received the formulae of the NAC for two limited cases, which are far away from the absorption threshold, Eq. (13), and close to the absorption threshold, Eq. (15). The formulae of the NAC contain a quantum number m characterizing confined phonons and easy to come back to the case of unconfined phonon Eq. (14) and Eq. (16). We numerically calculated and graphed the NAC for the GaAs/GaAsAl quantum well to clarify the theoretical results. The NAC depends very strongly on the quantum number m characterizing confined phonons, energy of EMW $\hbar\Omega$, amplitude E_o, width of quantum wells L, and temperature T of the system. There are more resonant peaks of the absorption coefficient appearing than in case of unconfined phonons and linear absorption [4]. The first one appears at $\Omega = \omega_o$, and the second one which appears at $\Omega = \omega_o$ is higher. When we consider case $E_o = 0$ in Eq. (16), the nonlinear results will turn back to linear results which were calculated by using the Kubo-Mori method [4]. There is only one resonant peak of the absorption coefficient appearing at $\Omega = \omega_0$. In short, the confinement of phonons in quantum wells makes the nonlinear absorption of a strong EMW by confined electrons much stronger.

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