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prof. Ing. Jindřich Petruška, CSc.
doc. Ing. Jiří Burša, Ph.D.
Ing. Alena Kruisová, Ph.D.
Ing. Tran Huu Nam, Ph.D.

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Hoang S.T., Marvalová B.: *Magneto-hyperelastic material in a uniform magnetic field: FEM Calculation of Stress and Strain*. Engineering Mechanics 2008, National Conference with International Participation. Svratka, May 12-15, 2008, Czech Republic, pp. 86-87, ISBN 978-80-87012-11-6.

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Hoang. S.T., Marvalová B.: *Constitutve Material Model of Fiberreinforced Composites in Comsol Multiphysics*. Technical Computing Prague 2007, 15th Annual Conference Proceedings. Prague, November 14, 2007, Czech Republic, pp. 53, ISBN 978-80-7080658-6.

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TECHNICAL UNIVERSITY OF LIBEREC
FACULTY OF MECHANICAL ENGINEERING

Ing. Hoang Sy Tuan

ELASTIC AND VISCOELASTIC BEHAVIOUR OF COMPOSITES WITH ELASTOMERIC MATRIX

ELASTICKÉ A VISKOELASTICKÉ CHOVÁNÍ KOMPOZITŮ S ELASTOMERICKOU MATRICÍ

Doctoral Dissertation

Supervisor:
Doc. Ing. Bohdana Marvalová, CSc
Technical University of Liberec

Liberec - 2010

Abstract

The viscous behavior of the fiber-reinforced composite materials with rubber-like matrix is modeled in the continuum mechanics framework by the Helmholtz free energy function and evolution equations of the internal variables. The decomposition of the free energy function and the chosen viscoelastic model are bases for formulation and description of the viscous characteristics of these anisotropic materials. Numerical simulations to predict the response of these materials in finite strains are performed.

The dissertation focuses on experimental evaluating the purely elastic and viscoelastic material parameters of proposed models via some standard experiments on relaxation, such as simple tension, pure shear and biaxial tensile tests. Both the isotropic and anisotropic materials were tested.

Several numerical examples were implemented in FEM software COMSOL Multiphysics and compared with the experimental results. The applications of the model were enlarged to predict other viscoelastic phenomena i.e. creep and influence of loading velocities on stresses. The influence of the directions of reinforcing fibers was also examined. The viscoelastic model was applied to a practical example that is an air-spring with two fiber reinforcements undergoing an internal pressure.

An extension of nonlinear theory for rubber-like anisotropic composites was applied to magneto-sensitive (MS) elastomers under an external magnetic field. The constitutive equations of both magnetic and mechanical fields were presented. Some numerical computations of a coupling of magnetic and mechanical problems were illustrated in order to describe a nonlinear characteristic of MS elastomer.

Key words:

Composites, rubber-like matrix, fiber-reinforced, viscoelasticity, magneto-sensitive elastomers, experimental, FEM.

8. Literatures

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7. Conclusions, discussions and future perspectives

In this dissertation, the viscoelastic behavior of the fiber-reinforced elastomer has been studied. The viscous characteristics of the anisotropic composites were identified by the suitable free energy function and the chosen viscoelastic models. Herein, the generalized Maxwell element model was used in two approaches with either inelastic strains or overstresses playing a role of internal variables.

Some standard experiments such as simple tensile, pure shearing and biaxial tensile tests for isotropic rubber-like materials and composite elastomers reinforced by two families of fibers under many relaxation stages were carried out. The non-contact optical stereo-correlation technique was used to determine precisely for experimental measurements of large deformations and evaluation of strains. The evaluation results were in good agreement with experimental data.

The implementation of the set of constitutive equations and evolution equations into a finite element program, Comsol Multiphysics, was established for modeling viscoelastic behaviour of both hyperelastic isotropic and anisotropic composites. The ability of the model to predict nonlinear viscoelastic behavior of isotropic and anisotropic materials was examined by comparing the theory to experimental results. Several examples relevant to viscoelastic responses, for instance the influence of the loading velocities, one- or multi- step relaxations and a creep were presented. More simulations of complicated boundary value problems of an air-spring tube with two fiber reinforcement were performed using the finite element method. The comparison between two approaches in overstress and inelastic strain variables was considered, this is just the initial step towards the nonlinear approach in inelastic strain variables.

The remaining task of the study was to develop a formulation of constitutive equations for anisotropic MS elastomers. We implemented several numerical solutions of simple boundary problems of nonlinear magneto-mechanical response of a body made of isotropic or anisotropic magnetosensitive elastomer subjected to a static magnetic field. The finite element software used proved a flexibility and ability of an easy implementation of fairly complicated coupled problem. The FE simulations involved not only the edge effects due to the finite geometry of the body but also the influence of the large displacement of the boundaries. The free energy functions that we have used are very simple forms and represent only a first approach towards a valuable constitutive model. Appropriate experiments which are in preparation will allow the elaboration of the constitutive relations. The constitutive model should involve also the complex dissipative (viscoelastic) behaviour of the material.

Contents

1. Introduction

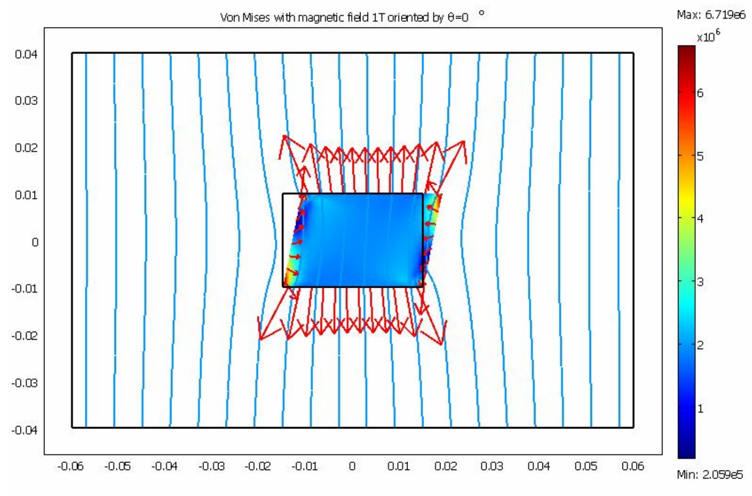
Fiber-reinforced elastomers (FREs) also well-known as anisotropic hyperelastic composites are widely used in practice, including industrial engineering, automobile, aircraft, biomechanics and medicine, for example gas pipes, automotive tyres, absorbers, belts, man-made (elastomeric) composites, etc...

These composites have many potential advantages due to high specific stiffness and strength, good corrosion resistance and thermal insulation. The typical anisotropic behavior is often formed by a number of fiber cords (usually one or two fibers coincide at each point) which are systematically arranged in a rubber-like matrix material. However, these materials not only have a highly non-linear behavior and possess anisotropic mechanical properties but also exhibit viscoelastic material behavior. Furthermore particularly important to this behavior is the heating of the structure because of internal dissipation and the temperature dependence of the material parameters. Therefore the ability to accurately predict the mechanical behavior of these materials is an important technological problem that is still far from being completely understood.

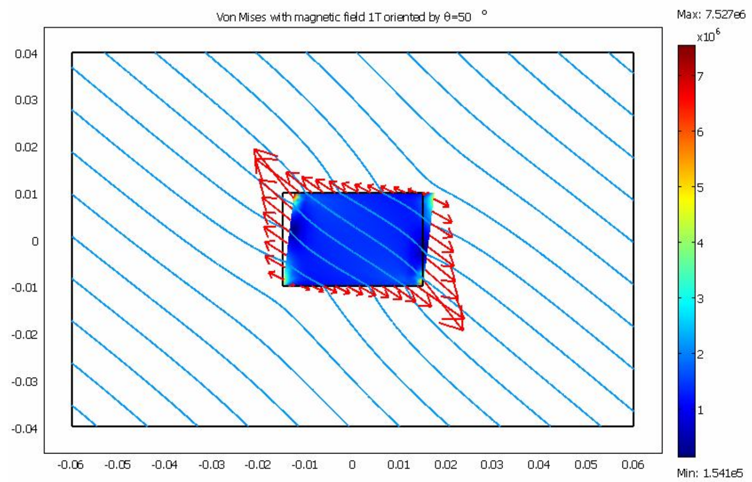
The main objective of the thesis is to identify and simulate the viscous characteristics of the fiber-reinforced composite materials with rubber-like matrix. The identification of the material parameters and the implementation of the numerical simulations that base on the chosen viscoelastic model are presented.

In this work both a mechanical experiment and a numerical simulation have been used in an effort to gain better insight into the mechanics causing the observed behavior and to facilitate ability performance of a viscoelastic model. There are many proposed viscoelastic models to deal viscoelastic problems of isotropic rubber-like materials as well as anisotropic hyperelastic composites with rubber-like matrix. However we focus on an approach in the continuum mechanical point of view.

In particular, to describe a viscoelastic behavior of anisotropic hyperelastic materials the existence of the Helmholtz free-energy functions is postulated. The free energy function is splitted into equilibrium and non-equilibrium parts governing the equilibrium (hyperelastic) and non-equilibrium (viscoelastic) responses, respectively. The non-equilibrium contribution of the free energy function depends not only on external variables, which are measurable and controllable quantities, but also on internal variables (hidden to the external observers). We use two approaches for the viscous response:

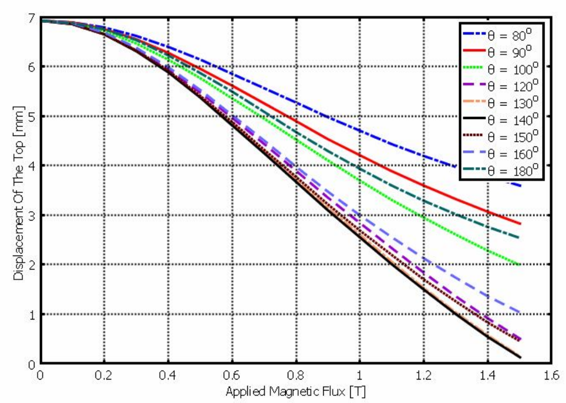


a) Magnetic field oriented in a vertical direction

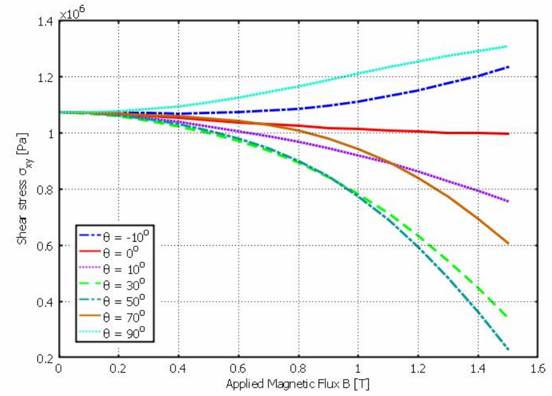


b) Magnetic orientation compared to a vertical direction $\theta=50^0$

Figure 26 – Simple shear state of the plate with different magnetic directions



a) Displacement of the top surface versus different directions of the applied magnetic field



b) Shear stress at the center of the body versus different directions of the applied magnetic field

Figure 27 – Dependencies of displacement and shear stress on the magnetic field

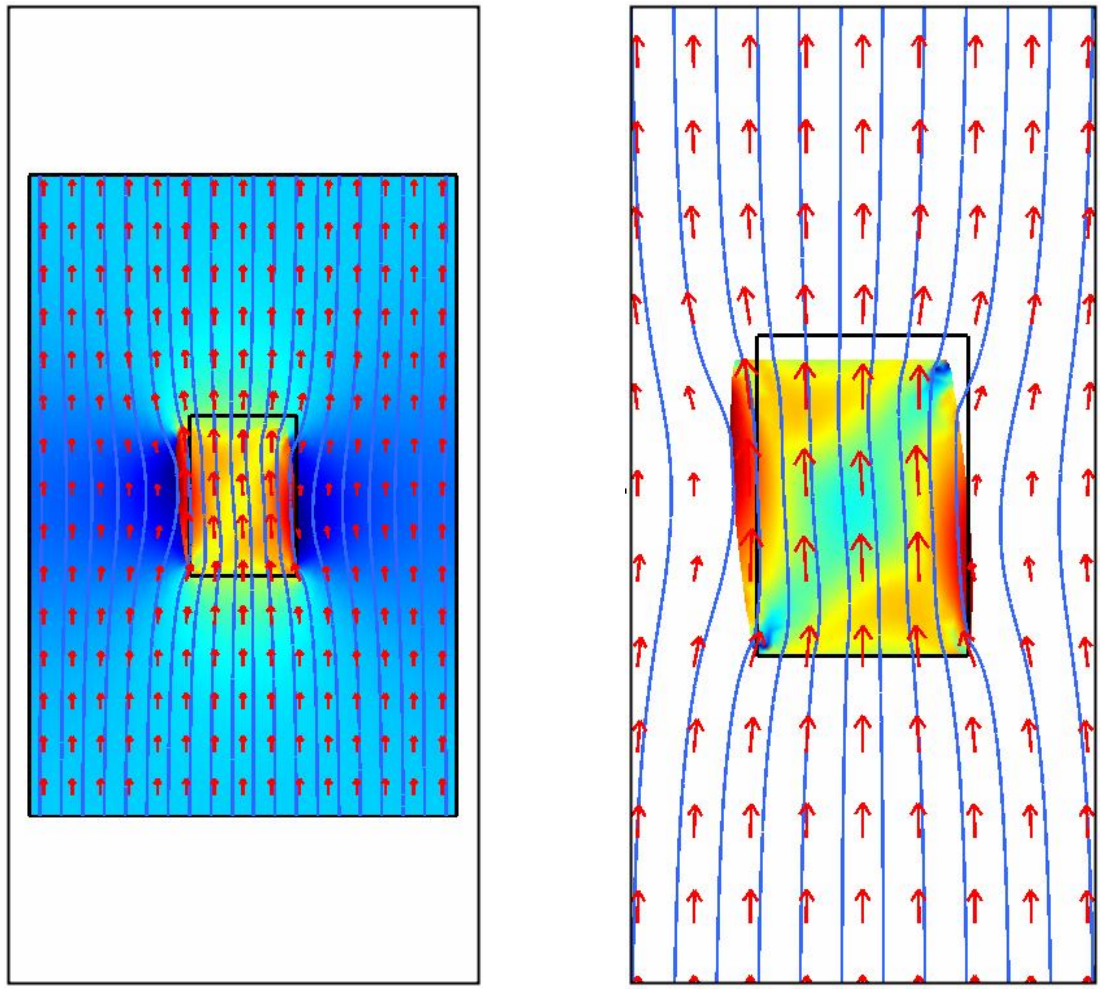
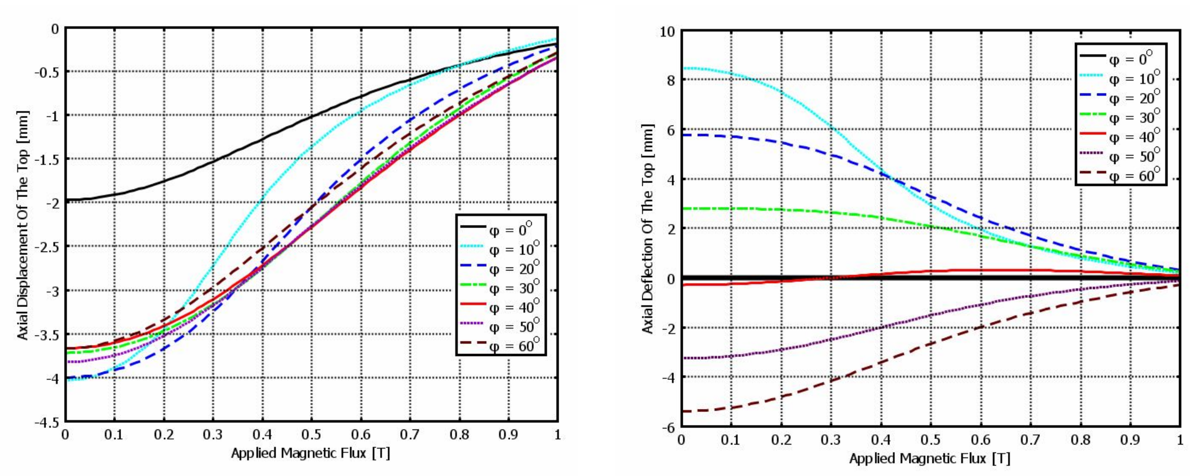


Figure 24 – Distribution of the magnetic field and the magnetization of the MS anisotropic material



a) Vertical displacement b) Horizontal displacement

Figure 25 – Displacements of the top surface of the block versus the magnetic flux density

- The first approach is formulated for internal stress-like variables (so called overstresses) and the formulation of the evolution equations is linear for loading close to thermodynamic equilibrium.
- The second approach with nonlinear evolution equations is formulated for internal strain-like variables (so called inelastic strains) by assuming parallel multiplicative decompositions of the deformation gradient into elastic and viscous parts.

We use rheological models such as Kelvin-Voigt or Maxwell models to establish evolution equations for internal variables.

Finally, we expand attention to develop constitutive formulations of anisotropic magneto-sensitive (MS) elastomer materials. Owing to the magnetic field is considered as a preferred direction in the reference configuration, hence the MS elastomers are subjected simultaneously to the action of the mechanical loading and magnetic field as similar to composites reinforced by fiber families. The theory of nonlinear magnetoelasticity for MS elastomers is applied to a number of simple boundary-value problems.

In order to achieve the above objectives, many tasks related to experimental and numerical FEM calculations should be implemented, namely some main tasks as follows:

- Propose the free energy functions used in the research.
- Formulate explicit expressions of equilibrium stress in deformation plane.
- Perform experiments in relaxation to measure the forces and the strains.
- Develop a Matlab program for evaluating the material parameters.
- Establish the viscoelastic model in FEM to calculate numerical simulations of viscoelastic materials.
- Extend constitutive equations of the anisotropic MS elastomers.
- Compute numerically some examples of MS elastomers in FEM.

2. Overview of literature

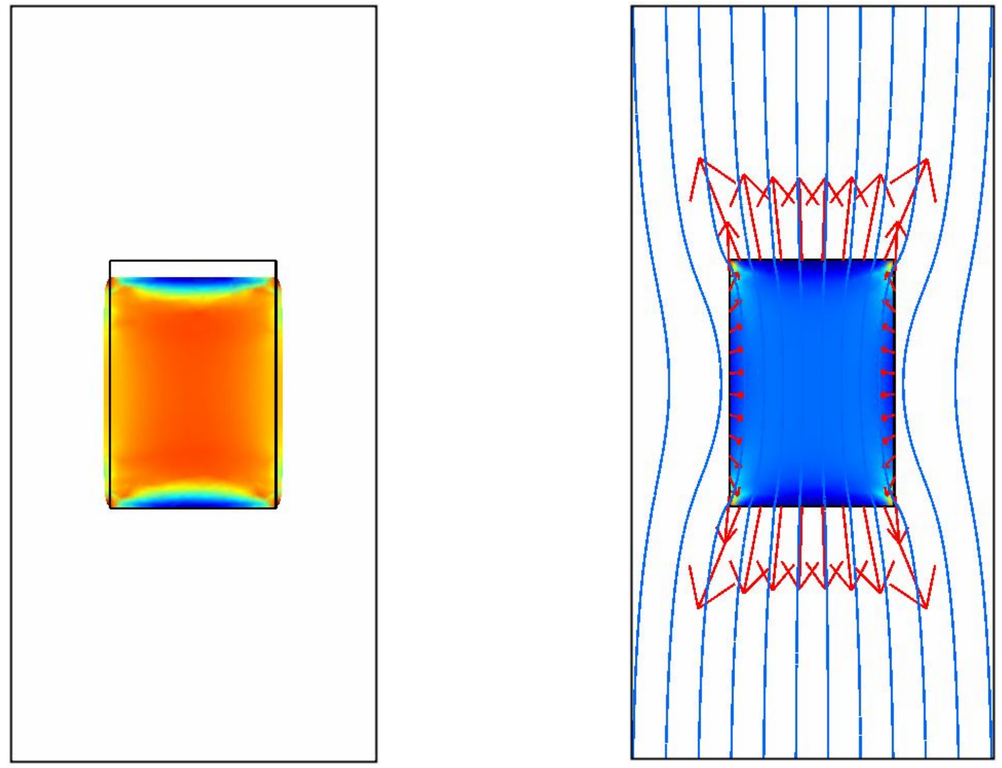
In this thesis, the viscoelastic behavior of the isotropic as well as anisotropic rubber-like materials is studied in the continuum mechanical theory by means of the free energy functions. The constitutive equation which interrelate the stress components and the strain components within a nonlinear regime can be found out, for example, Holzapfel (2000) or Truesdell & Noll (1992). The viscoelastic model of the anisotropic materials depends on the choice of internal variables and evolution equations. Evolution equations of overstresses proposed by Holzapfel & Gasser (2001) in the theory of linear viscoelasticity is quite simple to utilize for evaluating material parameters by experimental performances and implementing numerical simulations in FEM. However this model is believed in not credible the general problem of large deformations and large perturbations away from thermodynamic equilibrium, such as full thermo-mechanical coupling or high strain rates. Therefore, for this reason, the nonlinear viscoelastic model proposed by Nguyen et al (2007) is also given.

The constitutive formulation of magnetic and mechanical equations for MS elastomers is provided in series of recent studies by Brigadnov & Dorfmann (2003) and Dorfmann & Ogden (2003-2005). Specially, the influence of the magnetic field on the mechanical stress in the deforming body is incorporated through a magnetic stress tensor instead of through magnetic body forces included to the mechanical equilibrium equation, because the resulting total Cauchy stress tensor has the advantage of being symmetric, it can be referred to Dorfmann & Ogden (2004). The magnetic induction vector **B** and the magnetic field vector **H** are regarded as fundamental field variables and defined by the total free energy function.

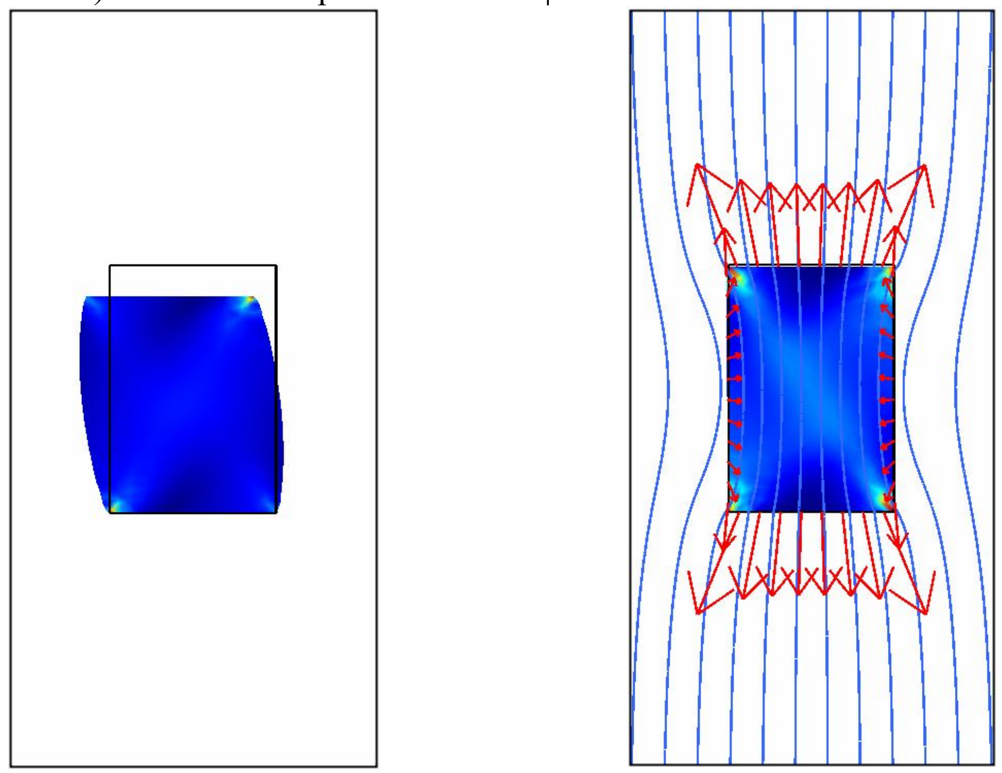
3. The decomposition of free energy function

The decomposition of the equilibrium part Ψ_{EQ} of free energy function within the isothermal regime is postulated to describe each contribution (volumetric, isotropic and anisotropic isochoric) which allows modeling an isotropic rubber-like material and a composite in which a rubber-like matrix material is reinforced by families of fibers. In all cases incompressible composite materials are assumed. The isotropic (isochoric) part of the free energy function is usually used classical models such as neo-Hookean, Mooney-Rivlin and Ogden models. To represent the anisotropic behaviour of the composite the anisotropic contribution of the free energy function can be chosen by either polynomial or exponential functions.

6.2. FEM solutions of MS anisotropic materials



a) Orientation of particle chains: $\varphi = 0^0$ at B = 0T and B = 1T



b) Orientation of particle chains: $\varphi = 30^0$ at B = 0T and B = 1T

Figure 23 – Deformation of the MS anisotropic block without and with a uniform magnetic field

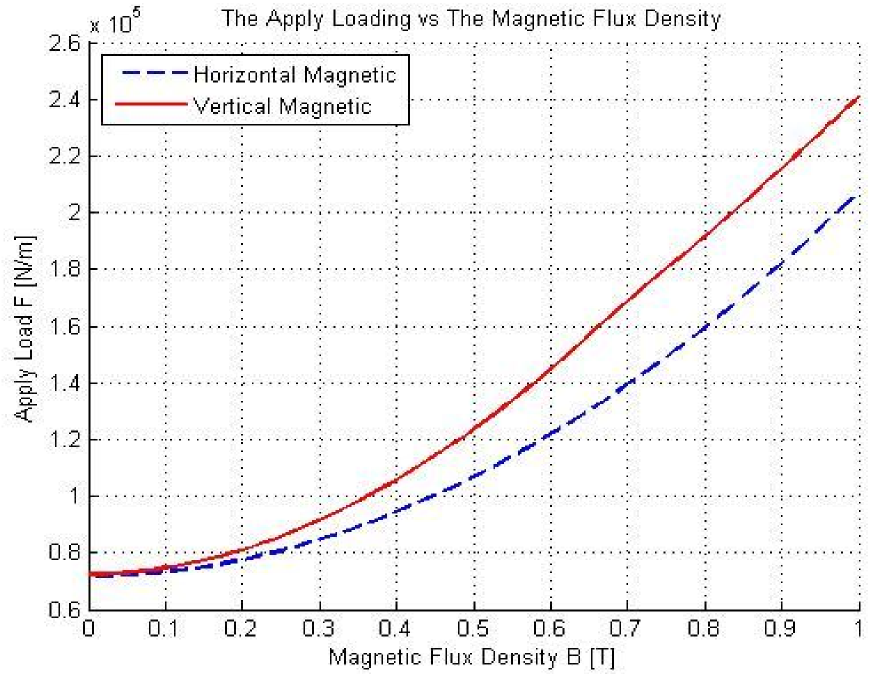


Figure 20 – Loading depends on direction and magnitude of magnetic field

a) b)

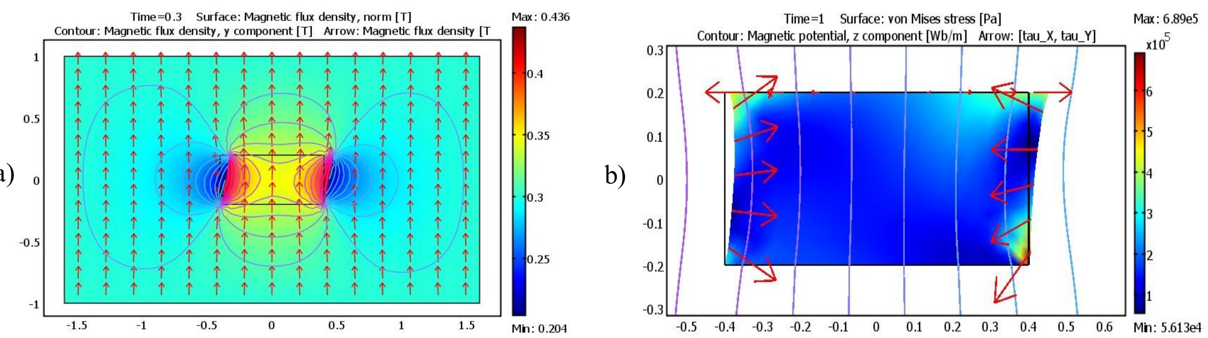


Figure 21 – MS Block in simple shear: a) Distribution of magnetic field,
b) Magnetic traction and Von Mises stress

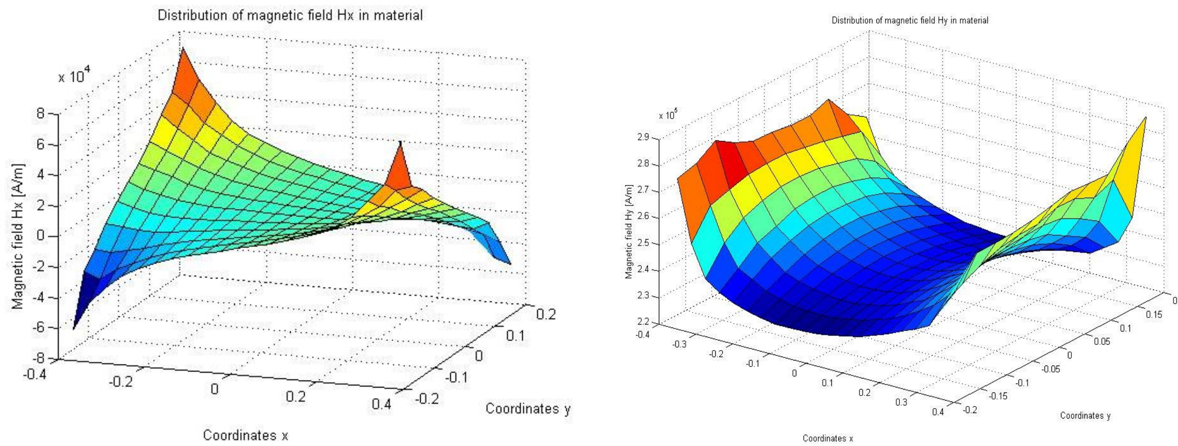


Figure 22 – Variation of magnetic fields H_x, H_y in the deformation body

4. Experiments and material parameter identification

For simplifying numerical estimations three assumptions are issued as follows

- Load is applied suddenly.
- Time in a relaxation process is long enough.
- Strain is unchanged throughout a relaxation process.

The elastic and viscoelastic parameters are evaluated by fitting experimental data by means of using linear and nonlinear least-square methods in Matlab software.

4.1. Isotropic composite materials

Evaluation of material parameters of the viscoelastic isotropic rubber-like materials is performed via basic experiments, namely simple tension, pure shearing and biaxial tensile tests.

Simple tension

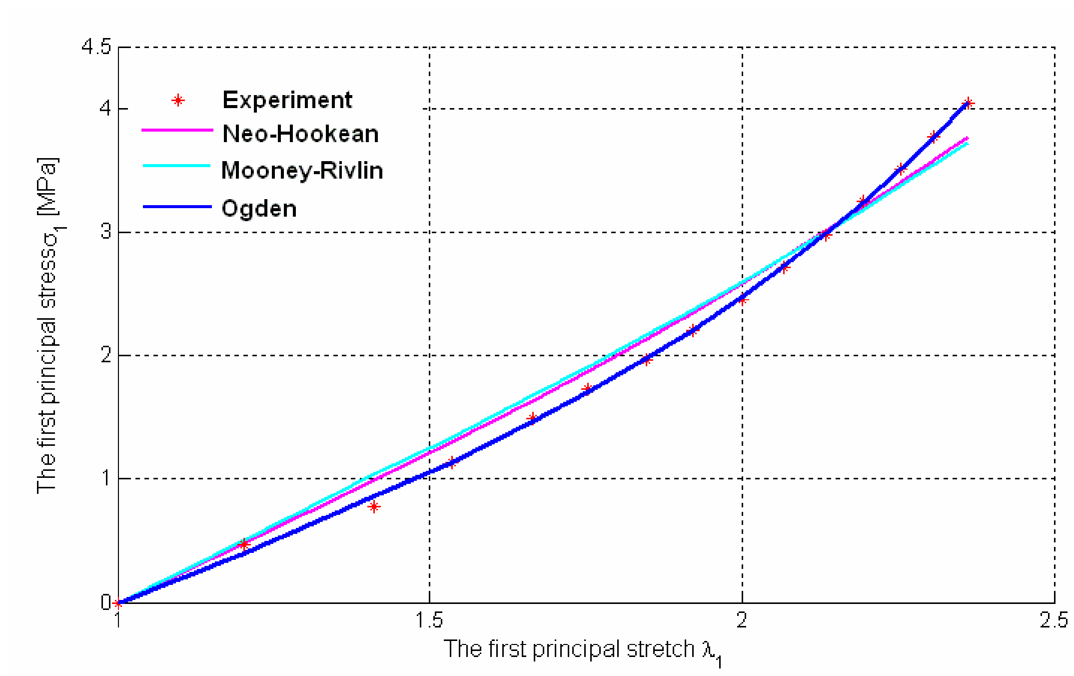


Figure 1 – Estimation of elastic coefficients of the isotropic material by a simple tensile test with different models

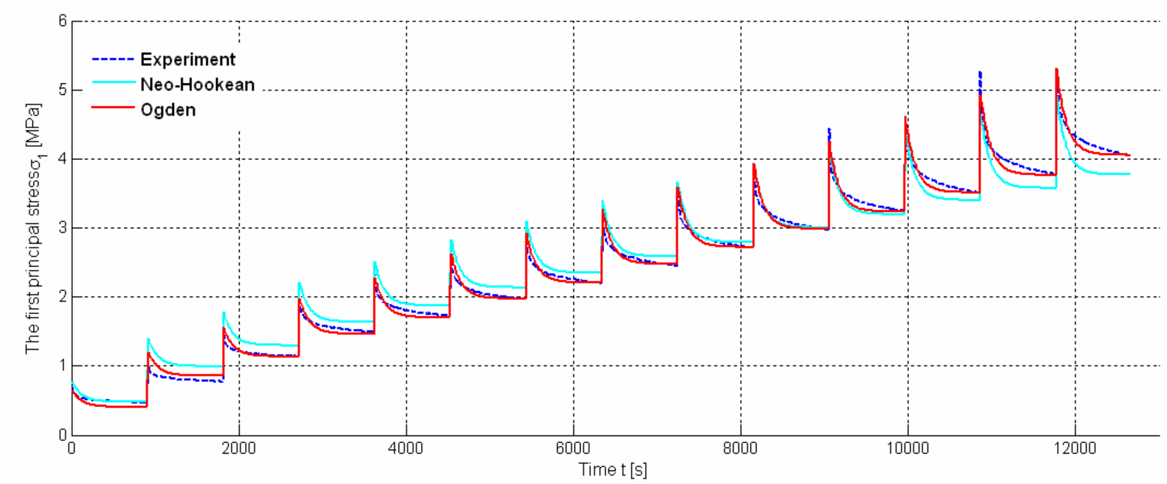


Figure 2 – Estimation of viscoelastic coefficients of the isotropic material by a simple tensile test with neo-Hookean and Ogden models

Table 1 – Elastic coefficients of the isotropic material by the simple tensile test

Model	Neo-Hookean	Mooney-Rivlin	Ogden
Material coefficients	c = 0,3678 MPa	c_1 = 0,3441 MPa c_2 = 0,0492 MPa	μ_1 = 0,5428 MPa; α_1 = 2,25 μ_2 = 0,00056 MPa; α_2 = 7,99
Shear modulus	μ = 0,7356 MPa	μ = 0,6883 MPa	μ = 0,6115 MPa

Table 2 – Viscoelastic coefficients of the isotropic material by the simple tensile test

Model	Neo-Hookean	Ogden
Relaxation time	$\tau = 114{,}2$ [s]	$\tau = 118{,}9$ [s]
Scale coefficients	$\beta = 0{,}278$	$\beta = 0{,}293$

Pure shearing test

Table 3 – Elastic coefficients of the isotropic material by the pure shearing test

Model	Neo-Hookean	Ogden's 2 parameters	Ogden's 4 parameters
Material coefficients	c = 0,2537 MPa	μ_1 = 0,3161 MPa α_1 = 2,84	μ_1 = -0,0001 MPa; α_1 = -15,41 μ_2 = 64,776 MPa; α_2 = 0,016
Shear modulus	μ = 0,5074 MPa	μ = 0,4483 MPa	μ = 0,5054 MPa

Table 4 – Viscoelastic coefficients of the isotropic material by the pure shearing test

Model	Neo-Hookean	Ogden
Relaxation time	$\tau = 135{,}4$ [s]	$\tau = 153{,}9$ [s]
Scale coefficients	$\beta = 0{,}16$	$\beta = 0{,}003$

6. Magneto-sensitive elastomer materials

We adopt the formulation of Dorfmann & Ogden (2003-2005) as the starting point. The general formulation of constitutive equations for anisotropic magnetoelastic interactions are based on Dorfmann & Ogden (2005) for both compressible and incompressible magnetoelastic materials.

The influence of the magnetic field on the mechanical stress in the deforming body may be incorporated through a magnetic stress tensor (see Dorfmann & Ogden, 2005).

For incompressible MS elastomers the volumetric component of the free energy function is chosen in the form

$$\Psi_{vol} = -p\left(J-1\right) \tag{1}$$

where p is the hydrostatic pressure.

In order to simulate behaviors of the incompressible magnetoelastic elastomer, we refer and inherit a simple form of the free energy function as proposed in Dorfmann's paper (2005). The isotropic and anisotropic contributions of the free energy function are used as follows

$$\Psi_{iso} = \frac{G}{4}\left[\left(1+\gamma\right)\left(\bar{I}_1-3\right)+\left(1-\gamma\right)\left(\bar{I}_2-3\right)\right] \tag{2}$$

$$\Psi_{ani} = \frac{1}{\mu_0}\left(\alpha I_4+\beta\bar{I}_5\right) \tag{3}$$

or

$$\Psi_{ani} = \frac{1}{\mu_0}\left(\alpha I_4+\beta\bar{I}_5\right)+\frac{k}{2}\left(\bar{I}_7-1\right)^2 \tag{4}$$

here $\bar{I}_1, \bar{I}_2, I_4, \bar{I}_5, \bar{I}_7$ are invariants of the right Cauchy-Green **C** and the magnetic field **B**, $G = G_0\left(1+\eta_G I_4\right)$, G_0 is the field independent shear modulus and $k = k_0\left(1+\eta_k I_4\right)$ represents the anisotropic characteristic of MS elastomers.

6.1. FEM solutions of MS isotropic materials

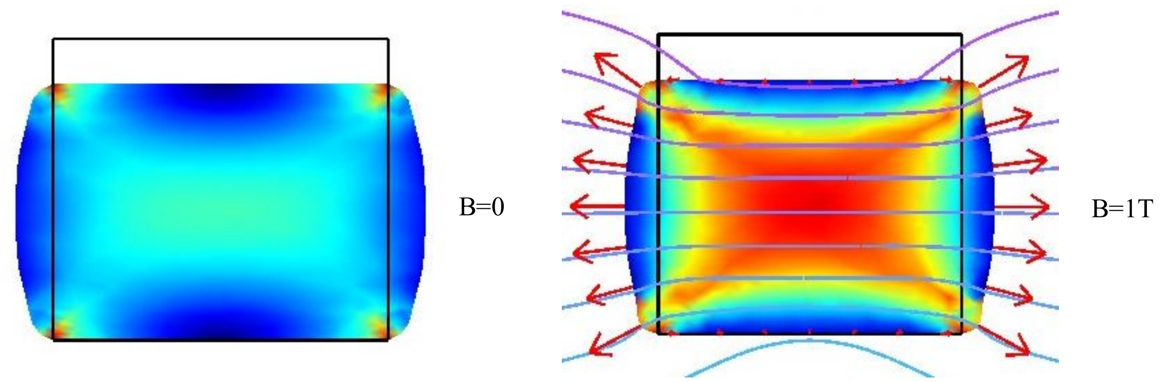
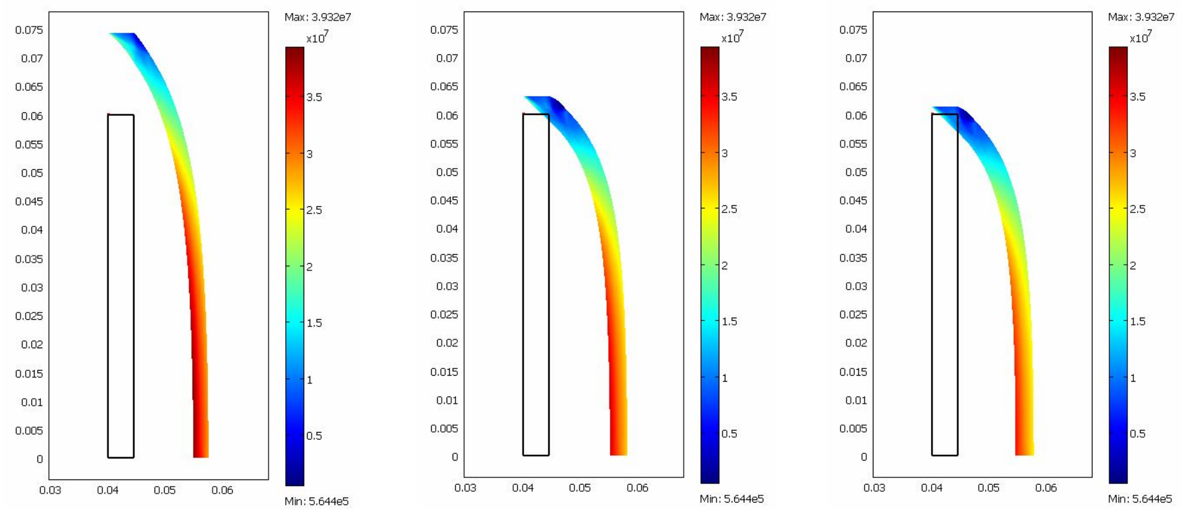


Figure 19 – Deformation of the block in horizontal magnetic field

Viscoelastic behavior of an air-spring



Static internal pressure Beginning of the creep The creep after 900s

Figure 16 – Deformation and stress of the tube at different time in a creep

5.3. Viscous responses of internal stress-like and strain-like variables

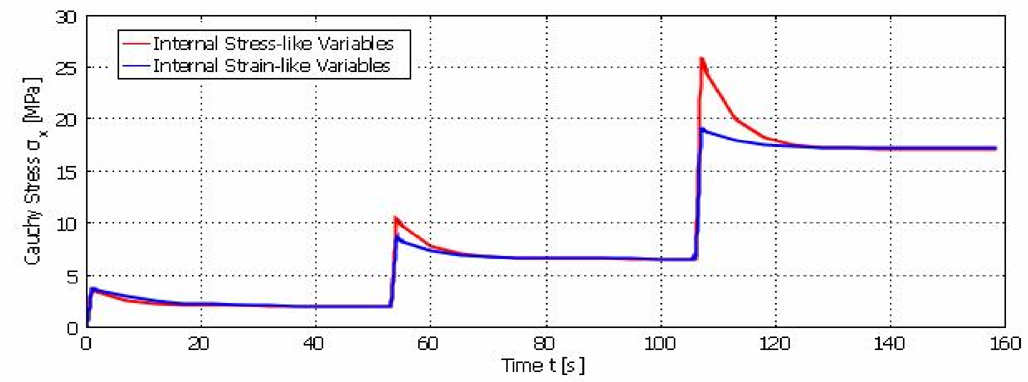


Figure 17 – Two approaches for the viscous response of the fiber-reinforced composite in relaxations

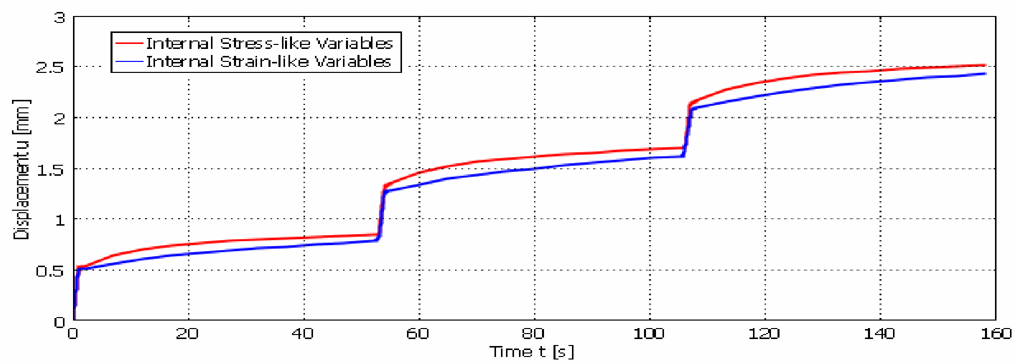


Figure 18 – Two approaches for the viscous response of the fiber-reinforced composite in creeps

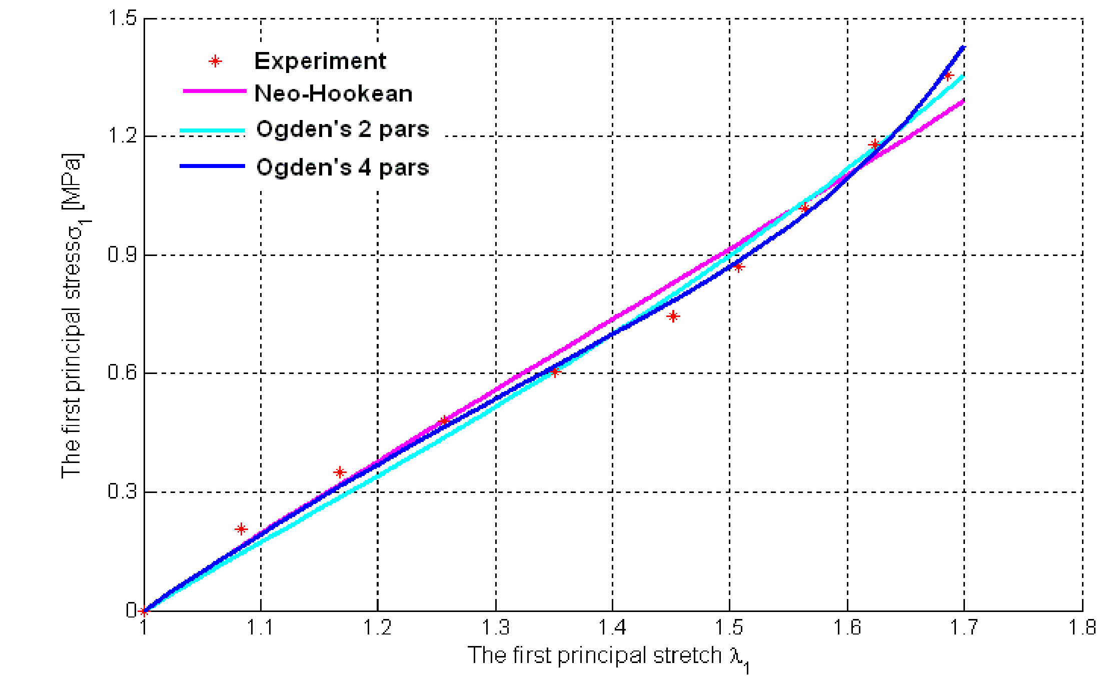


Figure 3 – Estimation of elastic coefficients of the isotropic material by a pure shearing test with different models

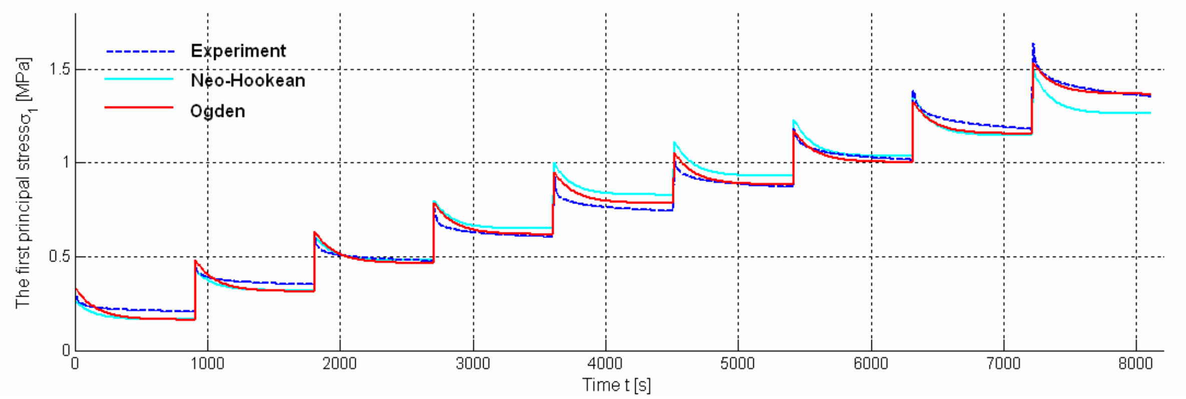


Figure 4 – Estimation of viscoelastic coefficients of the isotropic material by a pure shearing test with neo-Hookean and Ogden models

Biaxial tensile test

The evaluating results by biaxial tensile test are incredible due to the imperfect form of the specimen, specifically the effective cross-sectional area of the specimen arms. This effect can be eliminated by slits made in each of arms as recommended in Kuwabara et al (1998).

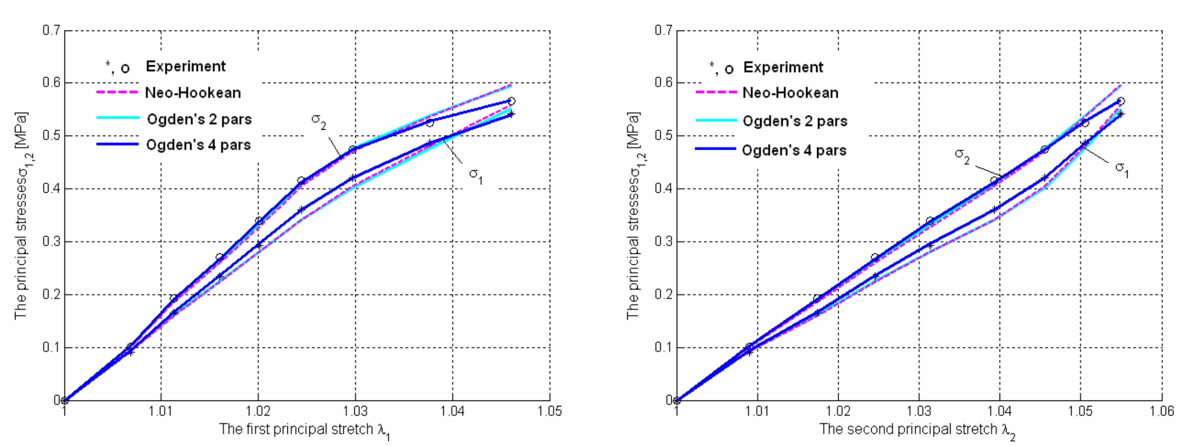


Figure 5 – Estimation of elastic coefficients of the isotropic material by a biaxial tensile test with different models

4.2. Fiber-reinforced composite materials

We implement experiments for composites reinforced with different fiber angles as 30^0, 40^0, 50^0 and 60^0 in multi-step relaxations, in which rectangular sheets with 30mm high and 4,5×220mm cross section.

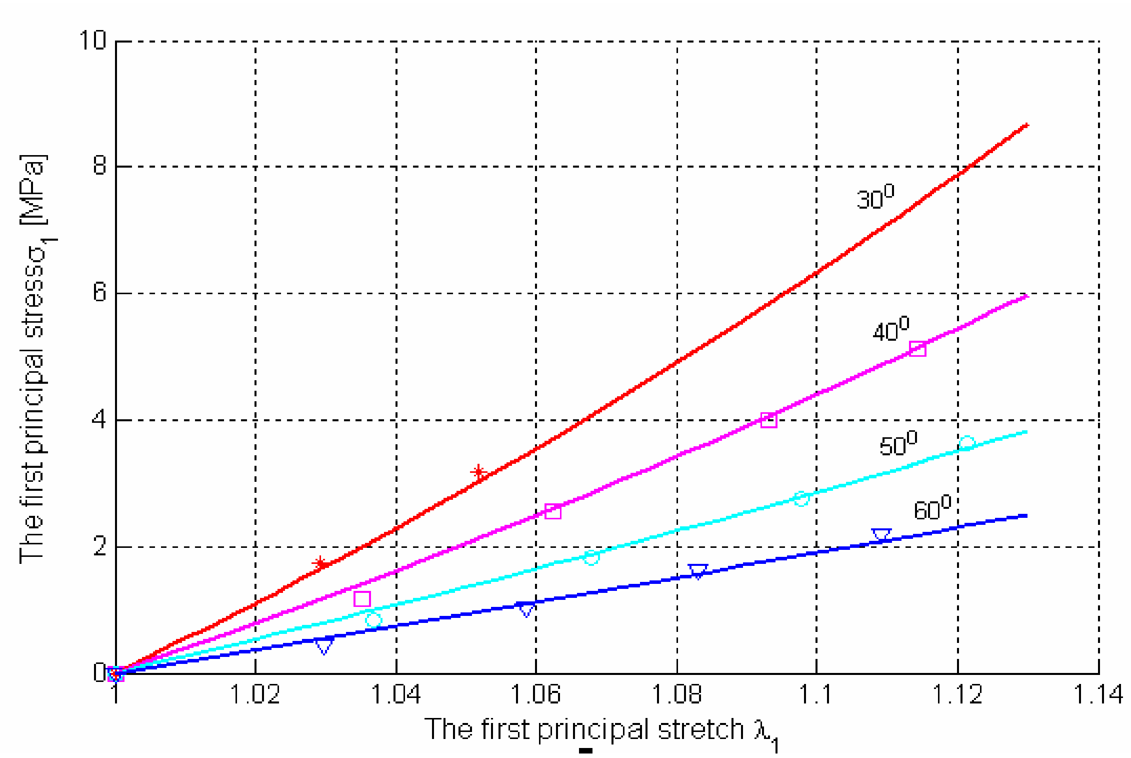


Figure 6 – Estimation of elastic coefficients of the composite reinforced with different fiber orientations by a pure shearing test

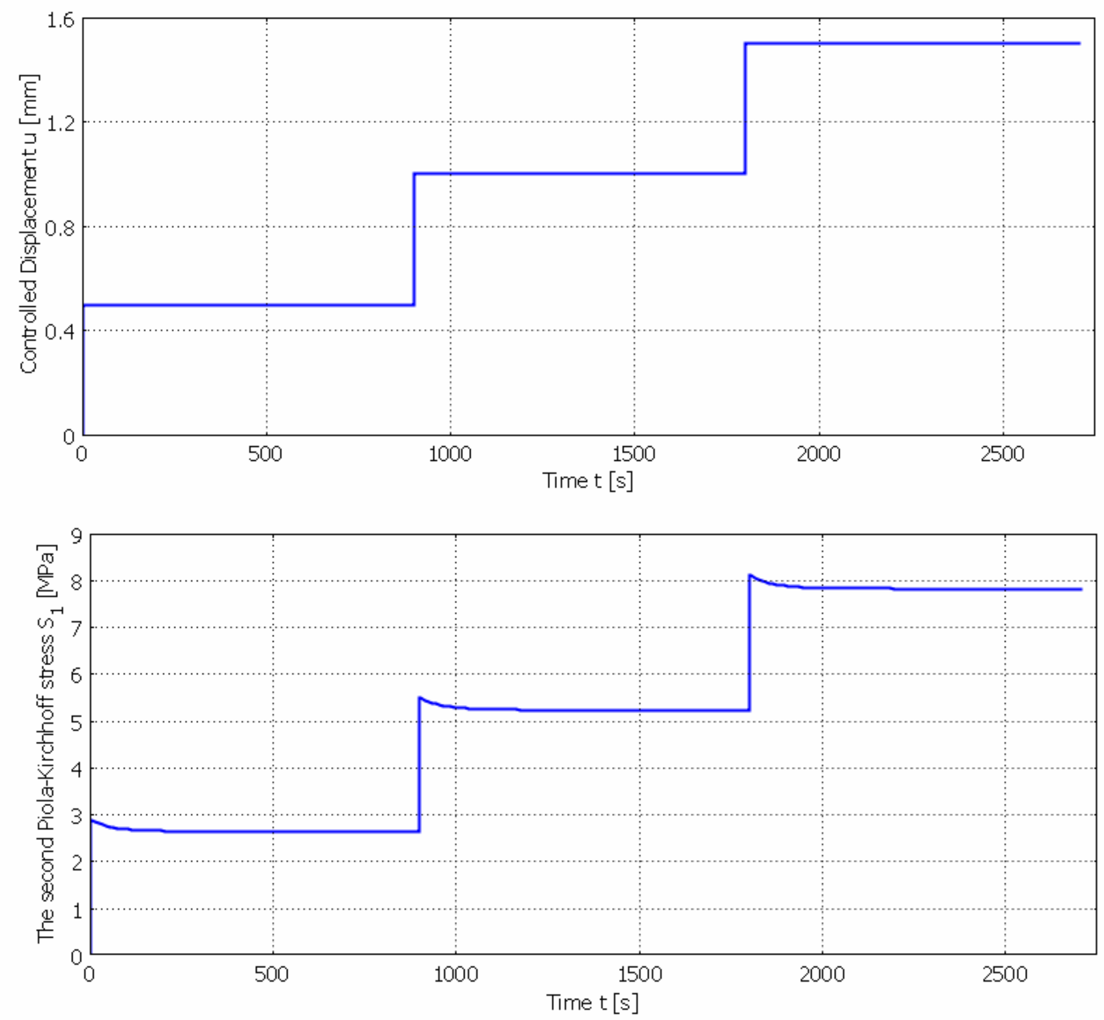


Figure 14 – Displacement and Piola-Kirchhoff stress of the anisotropic composite with fiber angles by 30^0 in relaxations

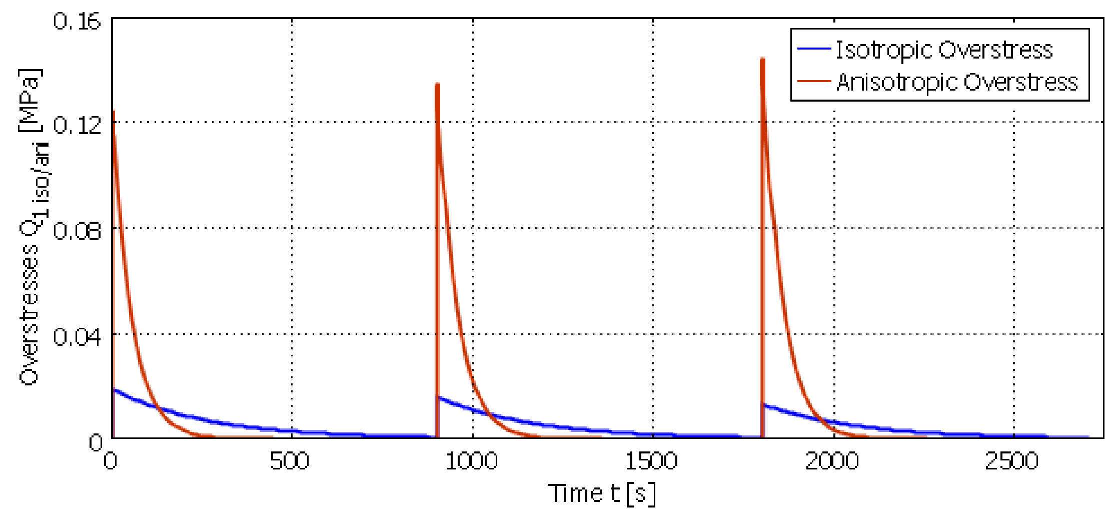


Figure 15 – The components of overstresses in the pure shear deformation

Prediction of creep process

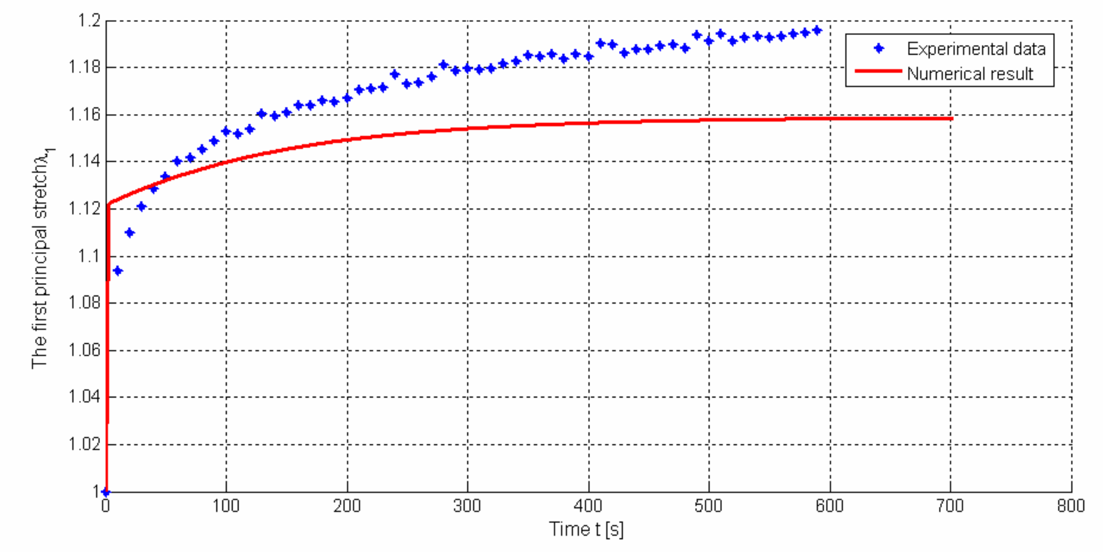


Figure 12 – Controlled force and displacement of simple tension in a creep

5.2. Fiber-reinforced composites

Equilibrium response of fiber-reinforced composite in pure shear

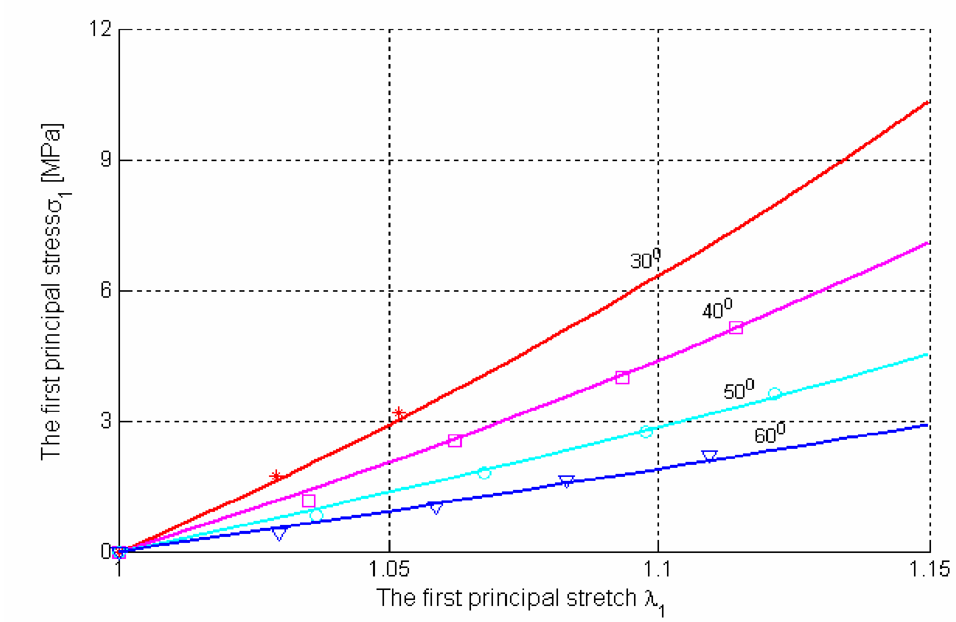
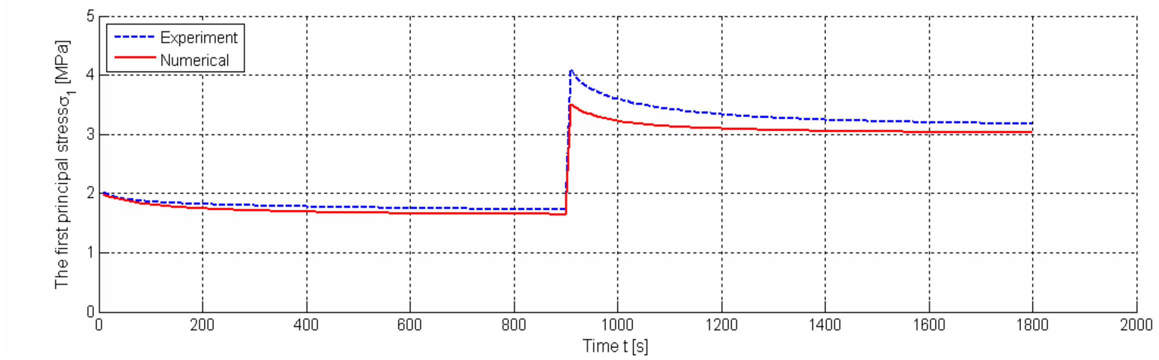
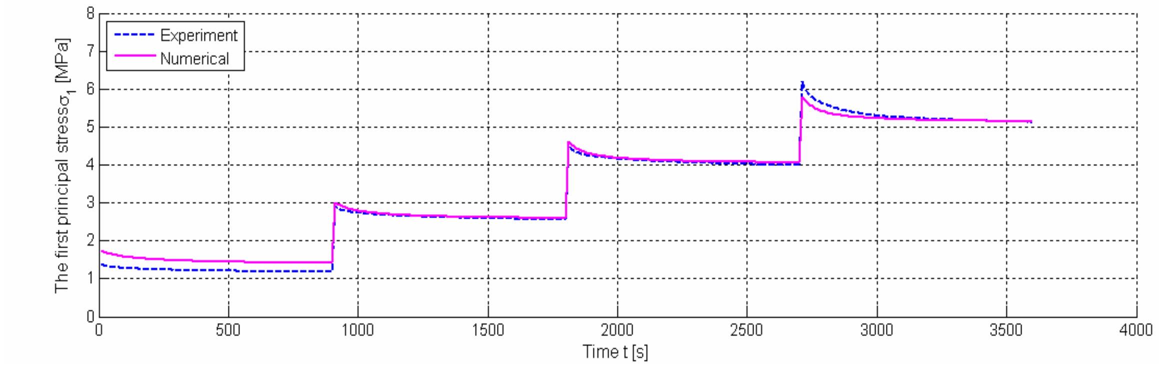


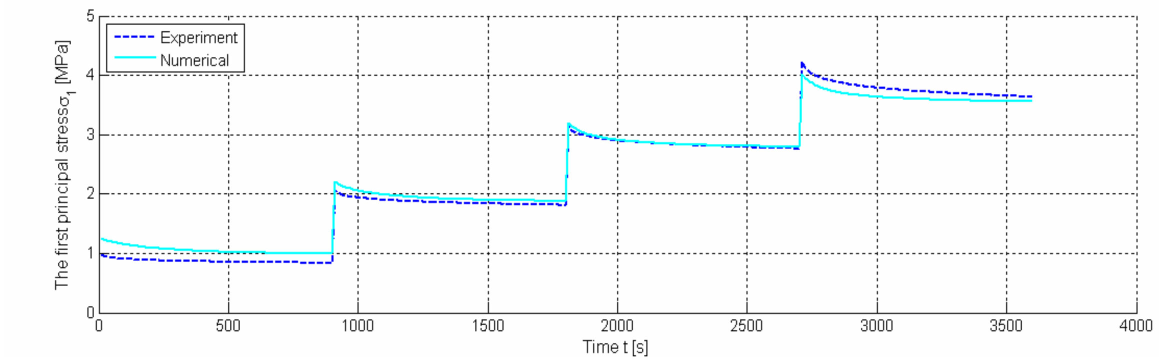
Figure 13 – Equilibrium Cauchy stress with different fiber directions in pure shear deformation (points denote experimental data, solid lines denote numerical results)



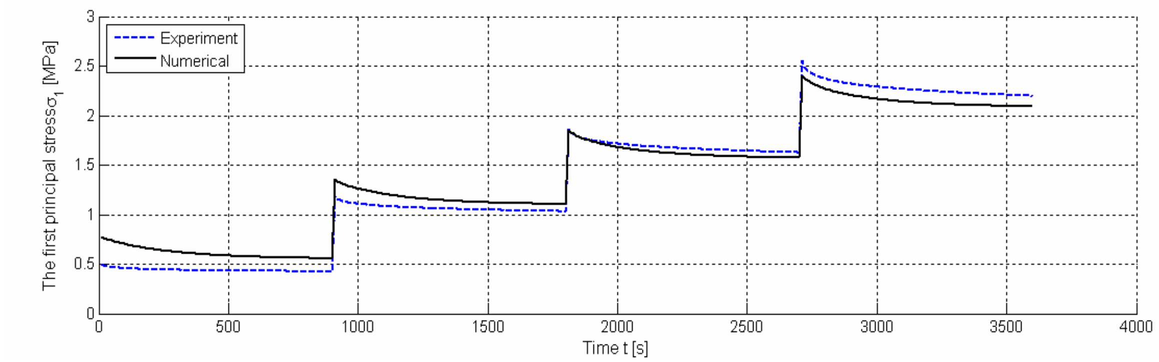
(a) Relaxation processes with fiber angle $\varphi = 30^0$



(b) Relaxation processes with fiber angle $\varphi = 40^0$



(c) Relaxation processes with fiber angle $\varphi = 50^0$



(d) Relaxation processes with fiber angle $\varphi = 60^0$

Figure 7 – Estimation of viscoelastic coefficients of the composite reinforced with different fiber orientations by a pure shearing test

5. Numerical simulations of viscoelastic composites

In this section we will represent some numeric simulations of hyperelastic as well as viscoelastic behavior of composite materials. The main goal is to verify the performance of constitutive viscoelastic models presented associating with the material parameters determined from the evaluation of experiments.

5.1. Isotropic (hyperelastic) rubber-like materials

Equilibrium stress-strain responses

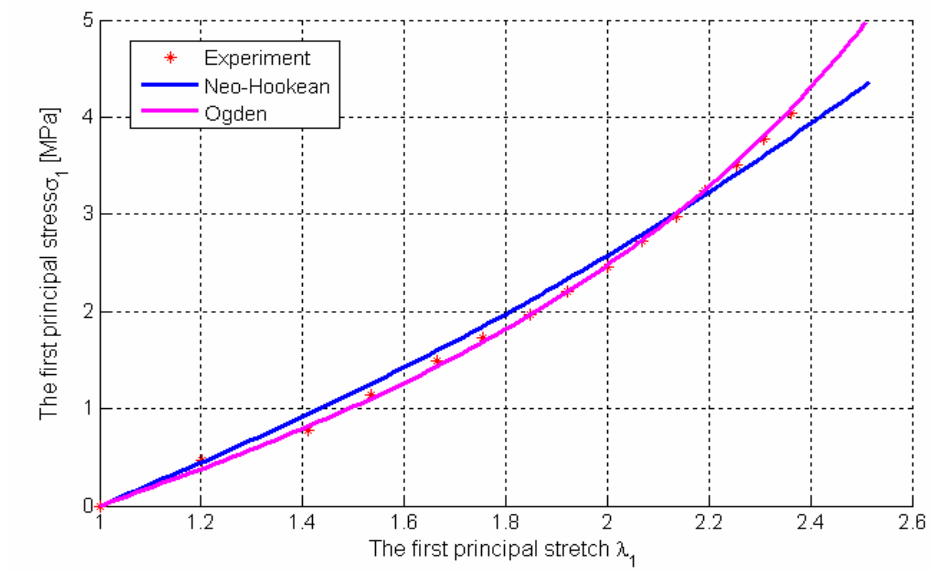


Figure 8 – First principal stress versus stretch of simple tension deformation

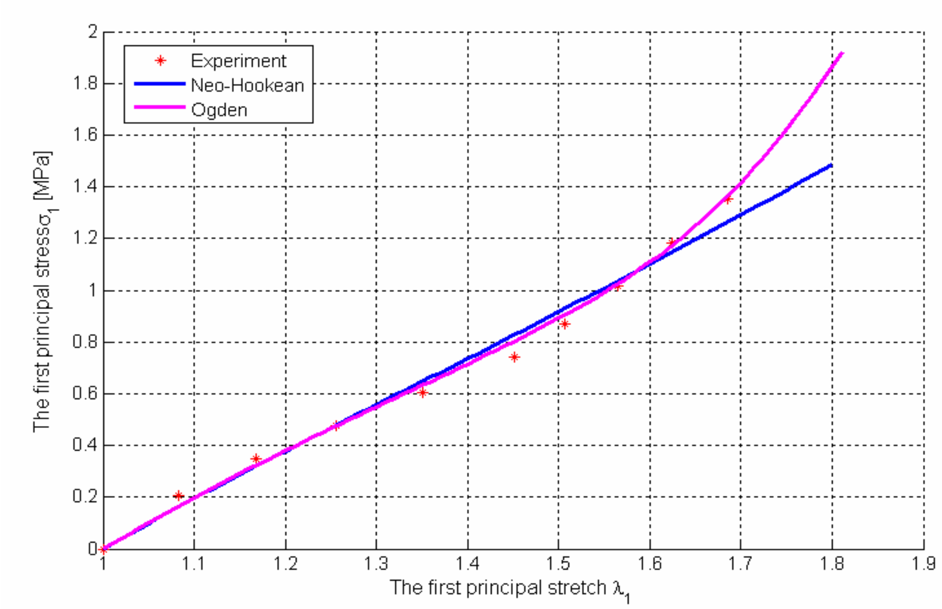


Figure 9 – First principal stress versus stretch of pure shear deformation

Viscoelastic behavior of isotropic materials

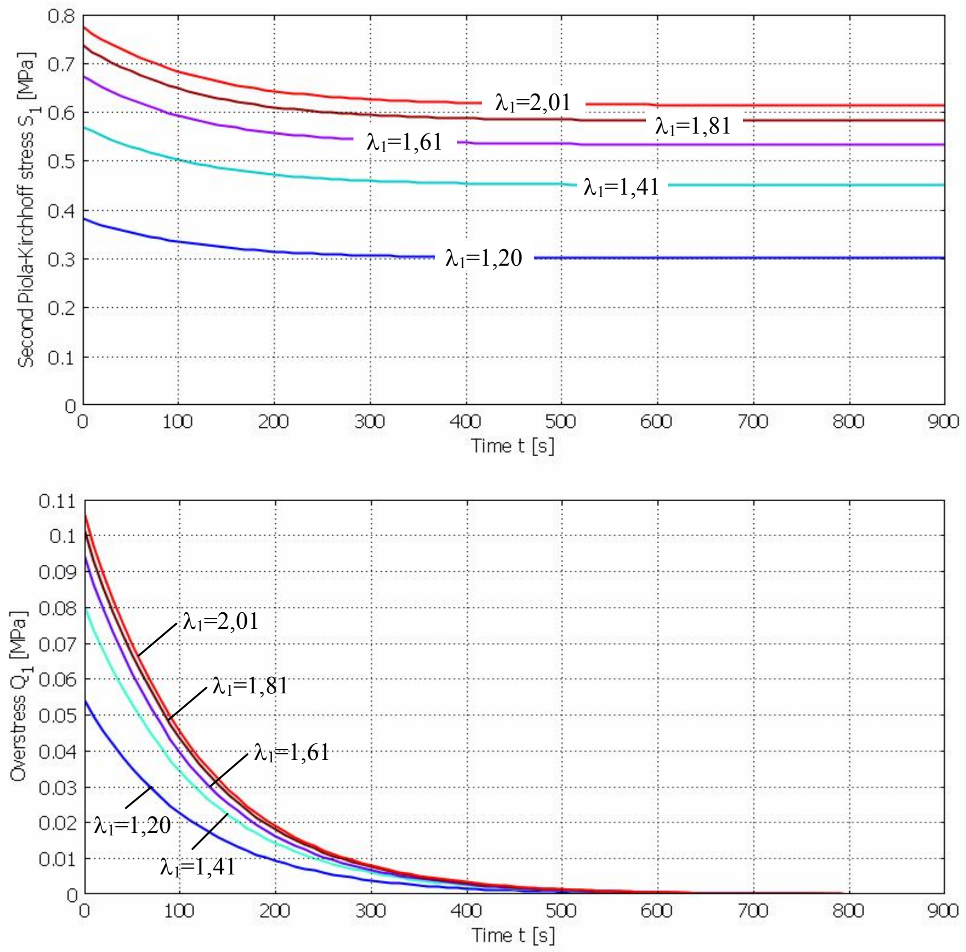
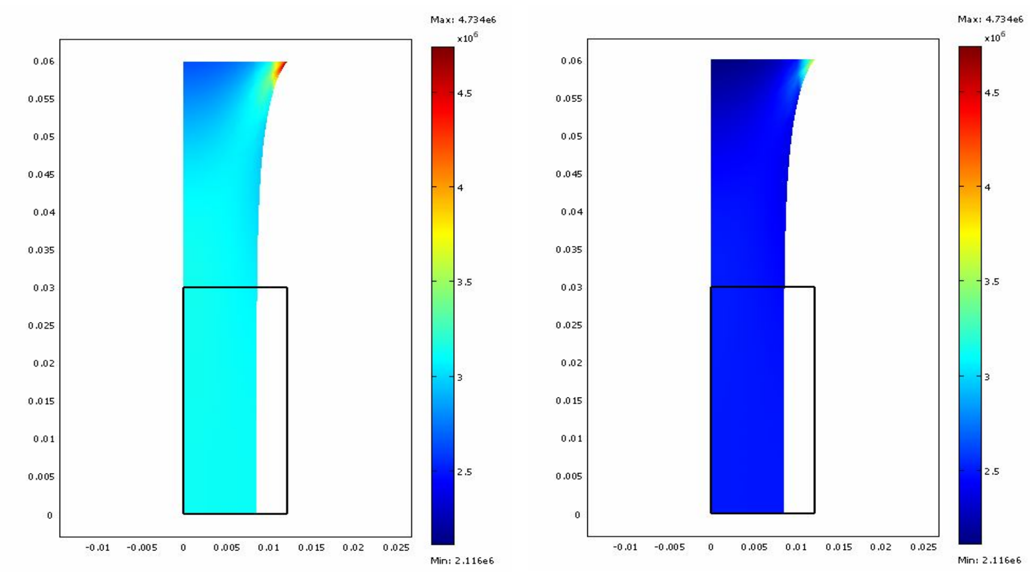


Figure 10 – The second Piola-Kirchhoff stresses and overstresses versus time at different stretches



$t=0.0^{+}$ $t=900s$

Figure 11 – Stress and deformation of simple tension in relaxation